LYAPUNOV STABILITY ANALYSIS OF DISCRETE-TIME SYSTEMS:

As in case of continuous-time systems, asymptotic stability is the most important concept in the stability of equilibrium states of discrete-time systems.

It may be noted that for discrete-time systems, we use the forward difference $V[x(K+1)T] - V[x(KT)] = \Delta V[x(KT)]$ in place of $\dot{x}$.

$$\Delta V[x(KT)] = V[x(K+1)T] - V[x(KT)].$$

<u>Theorem:</u>

Consider the discrete-time system

$\cancel{\Delta V[x(KT)]}$,

T → Samp

Suppose there exists a scalar function $V(x)$, continuous in x, such that

1. $V(x) > 0$ for $x \neq 0$
2. $\Delta V(x) < 0$ for $x \neq 0$
3. $V(0) = 0$
4. $V(x) \to \infty$ as $\|x\| \to \infty$

Then the equilibrium state $x = 0$ is asymptotically stable in the large and $V(x)$ is a Lyapunov function.

Lyapunov Stability Analysis of Linear Time Invariant Discrete-time systems:

Consider the discrete-time system described by

$$x(K+1) = Gx(K).$$

G is an $n \times n$ non-singular matrix.

Let us choose a possible Lyapunov Function

$$V(X(K)) = X^*(K) P X(K),$$

where P is a positive definite real symmetric matrix

Then

$$\Delta V(X(K)) = V(X(K+1)) - V(X(K))$$
$$= X^*(K+1) P X(K+1) - X^*(K) P X(K)$$
$$= [G X(K)]^* P [G X(K)] - X^*(K) P X(K)$$
$$= X^* G^* P G X(K) - X^*(K) P X(K)$$
$$= X^* (G^* P G - P) X(K).$$

Since $V(X(K))$ is chosen to be positive definite, we require for asymptotic stability that $\Delta V(X(K))$ be negative definite. Therefore,

$$\Delta V(X(K)) = -X^*(K) Q X(K),$$

where $Q = -(G^* P G - P)$ = Positive definite

Hence, for asymptotic stability of linear discrete-time systems, it is sufficient that Q be positive definite.

As in the case of continuous-time systems, it is convenient to specify first a real symmetric positive definite (Hermitian) matrix Q and then to see whether the matrix P determined from

$$G^* P G - P = -Q$$

is positive definite.