6 Synchronous motor

6.1 Principle of operation

In order to understand the principle of operation of a synchronous motor, let us examine what happens if we connect the armature winding (laid out in the stator) of a 3-phase synchronous machine to a suitable balanced 3-phase source and the field winding to a D.C source of appropriate voltage. The current flowing through the field coils will set up stationary magnetic poles of alternate North and South. (for convenience let us assume a salient pole rotor, as shown in Fig. 50). On the other hand, the 3-phase currents flowing in the armature winding produce a rotating magnetic field rotating at synchronous speed. In other words there will be moving North and South poles established in the stator due to the 3-phase currents i.e at any location in the stator there will be a North pole at some instant of time and it will become a South pole after a time period corresponding to half a cycle. (after a time $= \frac{1}{2f}$, where f = frequency of the supply). Let us assume that the stationary South pole in the rotor is aligned with the North pole in the stator moving in clockwise direction at a particular instant of time, as shown in Fig. 50. These two poles get attracted and try to maintain this alignment (as per lenz's law) and hence the rotor pole tries to follow the stator pole as the conditions are suitable for the production of torque in the clockwise direction. However the rotor cannot move instantaneously due to its mechanical inertia, and so it needs sometime to move. In the mean time, the stator pole would quickly (a time duration corresponding to half a cycle) change its polarity and becomes a South pole. So the force of attraction will no longer be present and instead the like poles experience a force

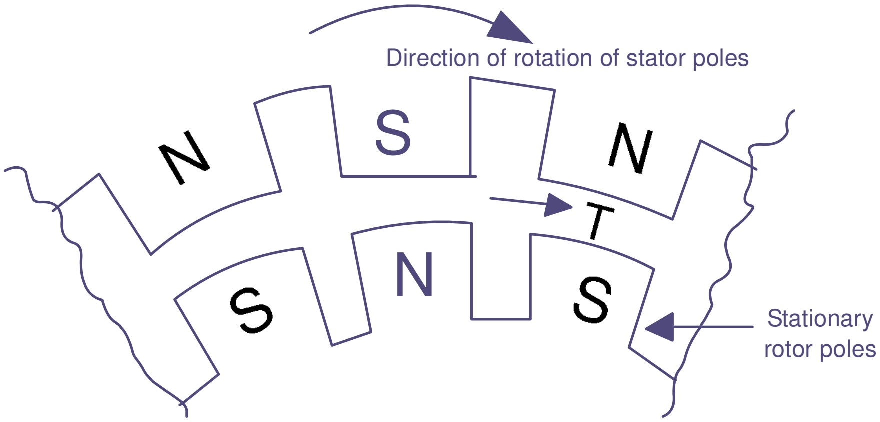


Figure 50: Force of attraction between stator poles and rotor poles - resulting in production of torque in clockwise direction

of repulsion as shown in Fig. 51. In other words, the conditions are now suitable for the

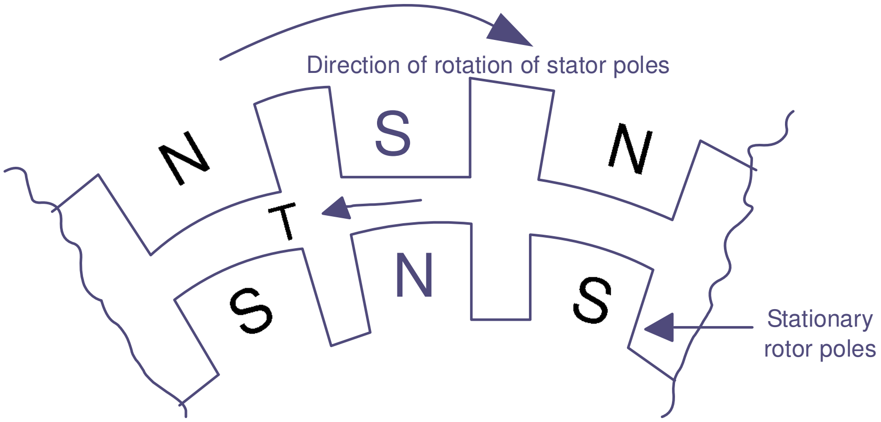


Figure 51: Force of repulsion between stator poles and rotor poles - resulting in production of torque in anticlockwise direction

production of torque in the anticlockwise direction. Even this condition will not last longer as the stator pole would again change to North pole after a time of $\frac{1}{2f}$. Thus the rotor will experience an alternating force which tries to move it clockwise and anticlockwise at twice the frequency of the supply, i.e. at intervals corresponding to $\frac{1}{2f}$ seconds. As this duration is quite small compared to the mechanical time constant of the rotor, the rotor cannot respond and move in any direction. The rotor continues to be stationary only.

On the contrary if the rotor is brought to near synchronous speed by some external means say a small motor (known as pony motor-which could be a D.C or AC induction rotor) mounted on the same shaft as that of the rotor, the rotor poles get locked to the unlike poles in the stator and the rotor continues to run at the synchronous speed even if the supply to the pony motor is disconnected.

Thus the synchronous rotor cannot start rotating on its own or usually we say that the synchronous rotor has no starting torque. So, some special provision has to be made either inside the machine or outside of the machine so that the rotor is brought to near about its synchronous speed. At that time, if the armature is supplied with electrical power, the rotor can pull into step and continue to operate at its synchronous speed. Some of the commonly used methods for starting synchronous rotor are described in the following section.

6.2 Methods of starting synchronous motor

Basically there are three methods that are used to start a synchronous motor:

- To reduce the speed of the rotating magnetic field of the stator to a low enough value that the rotor can easily accelerate and lock in with it during one half-cycle of the rotating magnetic field's rotation. This is done by reducing the frequency of the applied electric power. This method is usually followed in the case of inverter-fed synchronous motor operating under variable speed drive applications.
- To use an external prime mover to accelerate the rotor of synchronous motor near to its synchronous speed and then supply the rotor as well as stator. Ofcourse care should be taken to ensure that the direction of rotation of the rotor as well as that of the rotating magnetic field of the stator are the same. This method is usually followed in the laboratory- the synchronous machine is started as a generator and is then connected to the supply mains by following the synchronization or paralleling procedure. Then the power supply to the prime mover is disconnected so that the synchronous machine will continue to operate as a motor.
- To use damper windings or amortisseur windings if these are provided in the machine. The damper windings or amortisseur windings are provided in most of the large synchronous motors in order to nullify the oscillations of the rotor whenever the synchronous machine is subjected to a periodically varying load.

Each of these methods of starting a synchronous motor are described below in detail.

6.2.1 Motor Starting by Reducing the supply Frequency

If the rotating magnetic field of the stator in a synchronous motor rotates at a low enough speed, there will be no problem for the rotor to accelerate and to lock in with the stator's magnetic field. The speed of the stator magnetic field can then be increased to its rated operating speed by gradually increasing the supply frequency f up to its normal 50- or 60-Hz value.

This approach to starting of synchronous motors makes a lot of sense, but there is a big problem: Where from can we get the variable frequency supply? The usual power supply systems generally regulate the frequency to be 50 or 60 Hz as the case may be. However, variable-frequency voltage source can be obtained from a dedicated generator only in the olden days and such a situation was obviously impractical except for very unusual or special drive applications.

But the present day solid state power converters offer an easy solution to this. We now have the rectifier- inverter and cycloconverters, which can be used to convert a constant frequency AC supply to a variable frequency AC supply. With the development of such modern solid-state variable-frequency drive packages, it is thus possible to continuously control the frequency of the supply connected to the synchronous motor all the way from a fraction of a hertz up to and even above the normal rated frequency. If such a variable-frequency drive unit is included in a motor-control circuit to achieve speed control, then starting the synchronous motor is very easy-simply adjust the frequency to a very low value for starting, and then raise it up to the desired operating frequency for normal running.

When a synchronous motor is operated at a speed lower than the rated speed, its internal generated voltage (usually called the counter EMF) $E_A = K\phi\omega$ will be smaller than normal. As such the terminal voltage applied to the motor must be reduced proportionally with the frequency in order to keep the stator current within the rated value. Generally, the voltage in any variable-frequency power supply varies roughly linearly with the output frequency.

6.2.2 Motor Starting with an External Motor

The second method of starting a synchronous motor is to attach an external starting motor (pony motor) to it and bring the synchronous machine to near about its rated speed (but not exactly equal to it, as the synchronization process may fail to indicate the point of closure of the main switch connecting the synchronous machine to the supply system) with the pony motor. Then the output of the synchronous machine can be synchronised or paralleled with its power supply system as a generator, and the pony motor can be detached from the shaft of the machine or the supply to the pony motor can be disconnected. Once the pony motor is turned OFF, the shaft of the machine slows down, the speed of the rotor magnetic field B_R falls behind B_{net}, momentarily and the synchronous machine continues to operate as a motor. As soon as it begins to operates as a motor the synchronous motor can be loaded in the usual manner just like any motor.

This whole procedure is not as cumbersome as it sounds, since many synchronous motors are parts of motor-generator sets, and the synchronous machine in the motor-generator set may be started with the other machine serving as the starting motor. More over, the starting motor is required to overcome only the mechanical inertia of the synchronous machine without any mechanical load (load is attached only after the synchronous machine is paralleled to the power supply system). Since only the motor's inertia must be overcome, the starting motor can have a much smaller rating than the synchronous motor it is going to start. Generally most of the large synchronous motors have brushless excitation systems mounted on their shafts. It is then possible to use these exciters as the starting motors. For many medium-size to large synchronous motors, an external starting motor or starting by using the exciter may be the only possible solution, because the power systems they are tied to may not be able to handle the starting currents needed to use the damper (amortisseur) winding approach described next.

6.2.3 Motor Starting by Using damper (Amortisseur) Winding

As already mentioned earlier most of the large synchronous motors are provided with damper windings, in order to nullify the oscillations of the rotor whenever the synchronous machine is subjected to a periodically varying load. Damper windings are special bars laid into slots cut in the pole face of a synchronous machine and then shorted out on each end by a large shorting ring, similar to the squirrel cage rotor bars. A pole face with a set of damper windings is shown in Figure..

When the stator of such a synchronous machine is connected to the 3-Phase AC supply, the machine starts as a 3-Phase induction machine due to the presence of the damper bars, just like a squirrel cage induction motor. Just as in the case of a 3-Phase squirrel cage induction motor, the applied voltage must be suitably reduced so as to limit the starting current to the safe rated value. Once the motor picks up to a speed near about its synchronous speed, the DC supply to its field winding is connected and the synchronous motor pulls into step i.e. it continues to operate as a Synchronous motor running at its synchronous speed.

6.3 Behavior of a synchronous motor

The behavior of a synchronous motor can be predicted by considering its equivalent circuit on similar lines to that of a synchronous generator as described below.

6.3.1 Equivalent circuit model and phasor diagram of a synchronous motor

The equivalent-circuit model for one armature phase of a cylindrical rotor three phase synchronous motor is shown in Fig. 52 exactly similar to that of a synchronous generator except that the current flows in to the armature from the supply. All values are given per phase. Applying Kirchhoff's voltage law to Fig. 52,

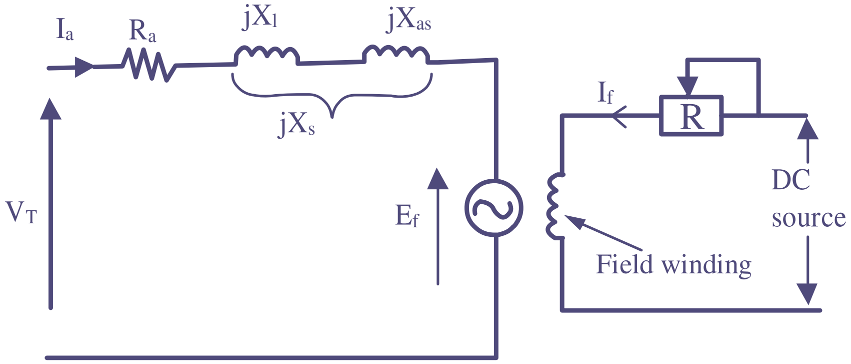


Figure 52: Equivalent-circuit model for one phase of a synchronous motor armature

$$\mathbf{V}_T = \mathbf{I}_a R_a + j\mathbf{I}_a X_l + j\mathbf{I}_a X_{as} + \mathbf{E}_f \tag{58}$$

Combining reactances, we have

$$X_s = X_l + X_{as} \tag{59}$$

Substituting Eqn. 59 in Eqn. 58

$$\mathbf{V}_T = \mathbf{E}_f + \mathbf{I}_a(R_a + jX_s) \tag{60}$$

$$or \quad \mathbf{V}_T = \mathbf{E}_f + \mathbf{I}_a\mathbf{Z}_s \tag{61}$$

where:
R_a = armature resistance (Ω/phase)
X_l = armature leakage reactance (Ω/phase)
X_s = synchronous reactance (Ω/phase)
Z_s = synchronous impedance (Ω/phase)
V_T = applied voltage/phase (V)
I_a = armature current/phase(A)

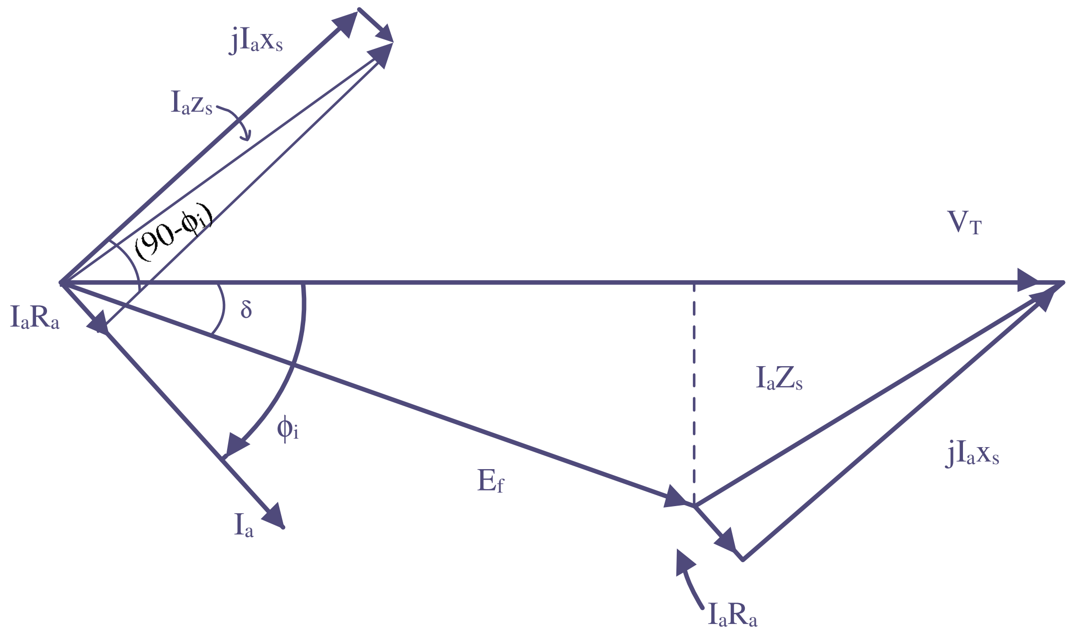


Figure 53: Phasor diagram corresponding to the equivalent-circuit model

A phasor diagram shown in Fig. 53, illustrates the method of determining the counter EMF which is obtained from the phasor equation;

$$\mathbf{E}_f = \mathbf{V}_T - \mathbf{I}_a \mathbf{Z}_s$$

The phase angle δ between the terminal voltage V_T and the excitation voltage E_f in Fig. 53 is usually termed the torque angle. The torque angle is also called the load angle or power angle.

6.3.2 Synchronous-motor power equation

Except for very small machines, the armature resistance of a synchronous motor is relatively insignificant compared to its synchronous reactance, so that Eqn. 61 to be approximated to

$$\mathbf{V}_T = \mathbf{E}_f + j\mathbf{I}_a X_s \tag{62}$$

The equivalent-circuit and phasor diagram corresponding to this relation are shown in Fig. 54 and Fig. 55. These are normally used for analyzing the behavior of a synchronous

motor, due to changes in load and/or changes in field excitation.
From this phasor diagram, we have,

$$I_a X_s \cos\theta_i = -E_f \sin\delta \tag{63}$$

Multiplying through by V_T and rearranging terms we have,

$$V_T I_a \cos\phi_i = \frac{-V_T E_f}{X_s} \sin\delta \tag{64}$$

Since the left side of Eqn. 64 is an expression for active power input and as the winding resistance is assumed to be negligible this power input will also represent the electromagnetic power developed, per phase, by the synchronous motor.
Thus,

$$P_{in,ph} = V_T I_a \cos\phi_i \tag{65}$$

or

$$P_{in,ph} = \frac{-V_T E_f}{X_s} \sin\delta \tag{66}$$

Thus, for a three-phase synchronous motor,

$$P_{in} = 3 * V_T I_a \cos\phi_i \tag{67}$$

or

$$P_{in} = 3 * \frac{-V_T E_f}{X_s} \sin\delta \tag{68}$$

Eqn. 66, called the synchronous-machine power equation, expresses the electro magnetic power developed per phase by a cylindrical-rotor motor, in terms of its excitation voltage and power angle. Assuming a constant source voltage and constant supply frequency, Eqn. 65 and Eqn. 66 may be expressed as proportionalities that are very useful for analyzing the behavior of a synchronous-motor:

$$P \propto I_a \cos\theta \tag{69}$$

$$P \propto E_f \sin\delta \tag{70}$$

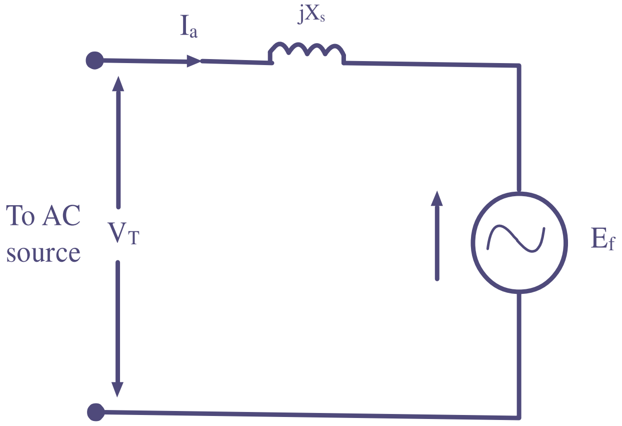


Figure 54: Equivalent-circuit of a synchronous-motor, assuming armature resistance is negligible

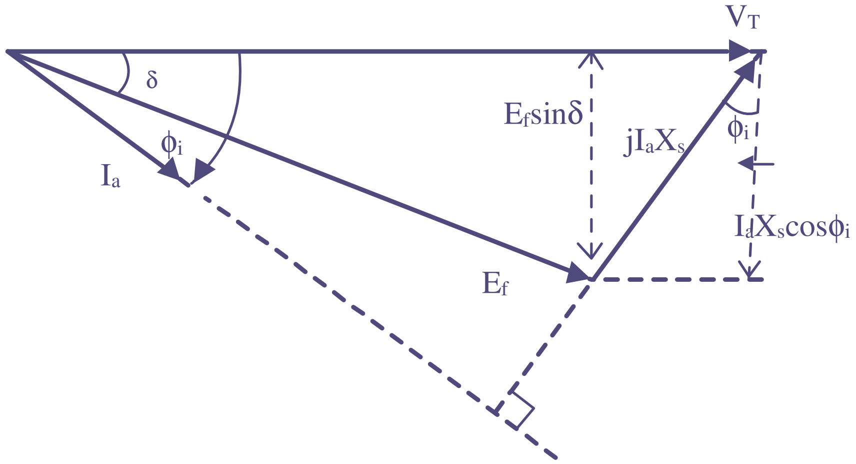


Figure 55: Phasor diagram model for a synchronous-motor, assuming armature resistance is negligible

6.3.3 Effect of changes in load on armature current, power angle, and power factor of synchronous motor

The effects of changes in mechanical or shaft load on armature current, power angle, and power factor can be seen from the phasor diagram shown in Fig. 56; As already stated, the applied stator voltage, frequency, and field excitation are assumed, constant. The initial load conditions, are represented by the thick lines. The effect of increasing the shaft load to twice its initial value are represented by the light lines indicating the new steady state conditions. These are drawn in accordance with Eqn. 69 and Eqn. 70, when the shaft load is doubled both $I_a \cos\phi_i$ and $E_f \sin\delta$ are doubled. While redrawing the phasor diagrams to show new steady-state conditions, the line of action of the new jI_aX_s phasor must be perpendicular to the new I_a phasor. Furthermore, as shown in Fig. 56, if the excitation is not changed, increasing the shaft load causes the locus of the E_f phasor to follow a circular arc, thereby increasing its phase angle with increasing shaft load. Note also that an increase in shaft load is also accompanied by a decrease in ϕ_i; resulting in an increase in power factor.

As additional load is placed on the machine, the rotor continues to increase its angle of lag relative to the rotating magnetic field, thereby increasing both the angle of lag of the counter EMF phasor and the magnitude of the stator current. It is interesting to note that during all this load variation, however, except for the duration of transient conditions

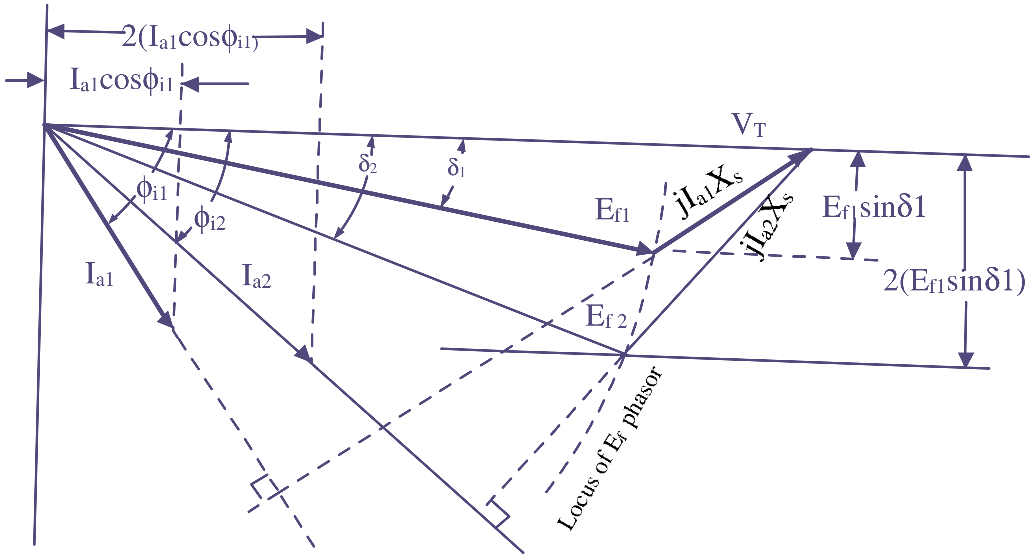


Figure 56: Phasor diagram showing effect of changes in shaft load on armature current, power angle and power factor of a synchronous motor

whereby the rotor assumes a new position in relation to the rotating magnetic field, the average speed of the machine does not change. As the load is being increased, a final point is reached at which a further increase in δ fails to cause a corresponding increase in motor torque, and the rotor pulls out of synchronism. In fact as stated earlier, the rotor poles at this point, will fall behind the stator poles such that they now come under the influence of like poles and the force of attraction no longer exists. Thus, the point of maximum torque occurs at a power angle of approximately 90° for a cylindrical-rotor machine, as is indicated by Eqn. 68. This maximum value of torque that causes a synchronous motor to pull out of synchronism is called the pull-out torque. In actual practice, the motor will never be operated at power angles close to 90° as armature current will be many times its rated value at this load.

6.3.4 Effect of changes in field excitation on synchronous motor performance

Intuitively we can expect that increasing the strength of the magnets will increase the magnetic attraction, and thereby cause the rotor magnets to have a closer alignment with the corresponding opposite poles of the rotating magnetic poles of the stator. This will obviously result in a smaller power angle. This fact can also be seen in Eqn. 68. When the shaft load is assumed to be constant, the steady-state value of $E_f \sin\delta$ must also be constant. An increase in E_f will cause a transient increase in $E_f \sin\delta$, and the rotor will accelerate. As the rotor changes its angular position, δ decreases until $E_f \sin\delta$ has the same steady-state value as before, at which time the rotor is again operating at synchronous speed, as it should run only at the synchronous speed. This change in angular position of the rotor magnets relative to the poles of rotating magnetic field of the stator occurs in a fraction of a second.

The effect of changes in field excitation on armature current, power angle, and power factor of a synchronous motor operating with a constant shaft load, from a constant voltage, constant frequency supply, is illustrated in Fig. 57. From Eqn. 69, we have for a constant shaft load,

$$E_{f1}\sin\delta_1 = E_{f2}\sin\delta_2 = E_{f3}\sin\delta_3 = E_f\sin\delta \tag{71}$$

This is shown in Fig. 57, where the locus of the tip of the E_f phasor is a straight line parallel to the V_T phasor. Similarly, from Eqn. 69, for a constant shaft load,

$$I_{a1}\cos\phi_{i1} = I_{a2}\cos\phi_{i2} = I_{a3}\cos\phi_{i3} = I_a\cos\phi_i \tag{72}$$

This is also shown in Fig. 57, where the locus of the tip of the I_a phasor is a line

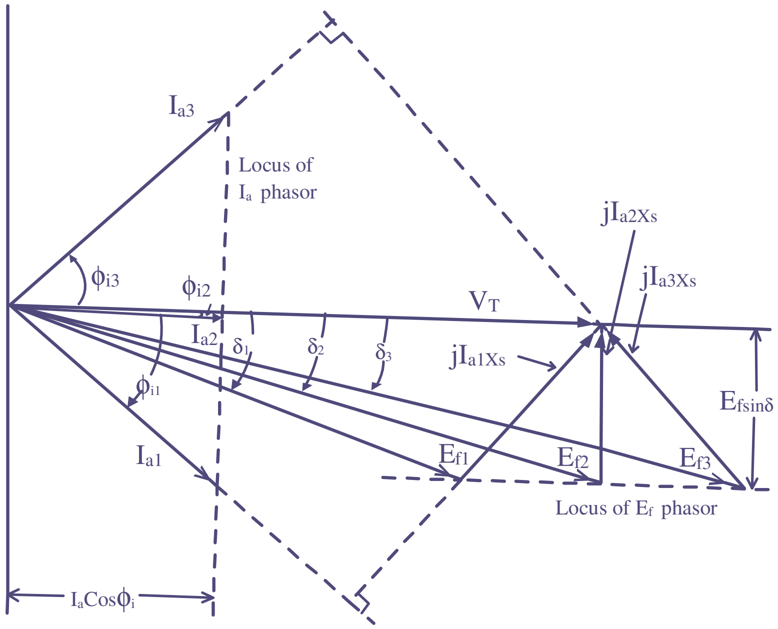


Figure 57: Phasor diagram showing effect of changes in field excitation on armature current, power angle and power factor of a synchronous motor

perpendicular to the V_T phasor.

Note that increasing the excitation from E_{f1} to E_{f3} in Fig. 57 caused the phase angle of the current phasor with respect to the terminal voltage V_T (and hence the power factor) to go from lagging to leading. The value of field excitation that results in unity power factor is called normal excitation. Excitation greater than normal is called over excitation, and excitation less than normal is called under excitation. Furthermore, as indicated in Fig. 57, when operating in the overexcited mode, $|E_f| > |V_T|$. In fact a synchronous motor operating under over excitation condition is sometimes called a synchronous condenser.

6.3.5 V curves

Curves of armature current vs. field current (or excitation voltage to a different scale) are called V curves, and are shown in Fig. 58 for typical values of synchronous motor loads. The curves are related to the phasor diagram in Fig. 57, and illustrate the effect of the variation of field excitation on armature current and power factor for typical shaft loads. It can be easily noted from these curves that an increase in shaft loads require an increase in field excitation in order to maintain the power factor at unity.

The locus of the left most point of the V curves in Fig. 58 represents the stability limit ($\delta = -90°$). Any reduction in excitation below the stability limit for a particular load will cause the rotor to pullout of synchronism.

The V curves shown in Fig. 58 can be determined experimentally in the laboratory by varying I_f at a constant shaft load and noting I_a as I_f is varied. Alternatively the V curves shown in Fig. 58 can be determined graphically by plotting $|I_a| vs. |E_f|$ from a family of phasor diagrams as shown in Fig. 57, or from the following mathematical expression for the V curves

$$\begin{aligned}(I_aX_s)^2 &= V_T^2 + E_f^2 - 2V_TE_f\cos\delta \\ &= V_T^2 + E_f^2 - 2V_TE_f\sqrt{1-\sin^2\delta} \\ &= V_T^2 + E_f^2 - 2\sqrt{V_T^2E_f^2 - V_T^2E_f^2\sin^2\delta}\end{aligned} \tag{73}$$

$$I_a = \frac{1}{X_s}.\sqrt{V_T^2 + E_f^2 - 2\sqrt{V_T^2E_f^2 - X_s^2.P_{in,ph}^2}} \tag{74}$$

Eqn. 74 is based on the phasor diagram and the assumption that R_a is negligible. It is to be noted that instability will occur, if the developed torque is less than the shaft load plus friction and windage losses, and the expression under the square root sign will be negative.

The family of V curves shown in Fig. 58 represent computer plots of Eqn. 74, by taking the data pertaining to a three-phase 10 hp synchronous motor i.e $V_{ph} = 230V$ and $X_s = 1.2\Omega/phase$.

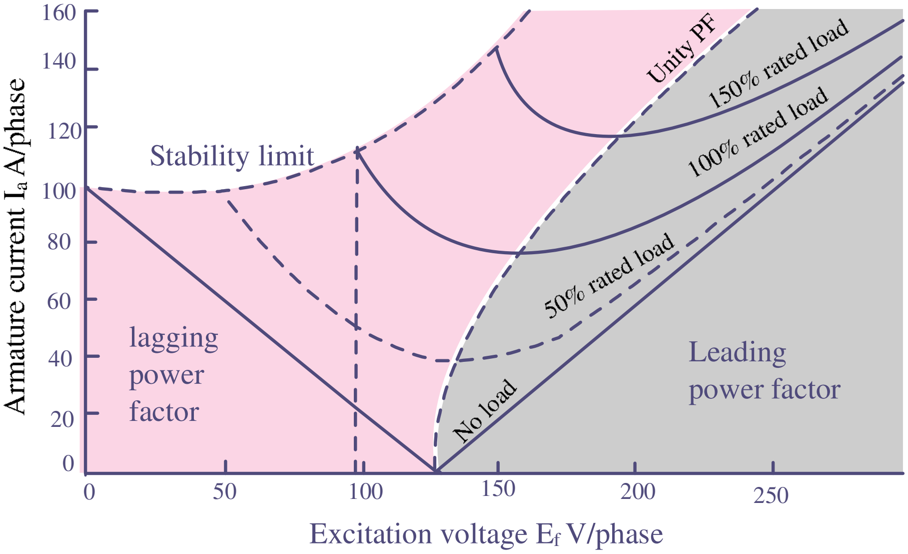


Figure 58: Family of representative V curves for a synchronous motor

6.3.6 Synchronous-motor losses and efficiency

The flow of power through a synchronous motor, from stator to rotor and then to shaft output, is shown in Fig. 59. As indicated in the power-flow diagram, the total power loss for the motor is given by

$$P_{loss} = P_{scl} + P_{core} + P_{fcl} + P_{f,w} + P_{stray}\ W \tag{75}$$

where:
P_{scl}= stator-copper loss
P_{fcl} = fie1d-copper.loss
P_{core} = core loss

$P_{f,w}$ = friction and windage loss
P_{stray} = stray load loss
Except for the transient conditions that occur when the field current is increased or decreased (magnetic energy stored or released), the total energy supplied to the field coils is constant and all of it is consumed as I^2R losses in the field winding. Just as in the case of the synchronous generator, the overall efficiency of a synchronous motor is given by

$$\eta = \frac{P_{shaft}}{P_{in} + P_{field}} = \frac{P_{shaft}}{P_{shaft} + P_{loss}} \tag{76}$$

Generally, the nameplates of synchronous motors and manufacturers' specification sheets customarily provide the overall efficiency for rated load and few load conditions only. Hence, only the total losses at these loads can be determined. The separation of losses into the components listed in Eqn. 75 needs a very involved test procedure in the laboratory. However, a closer approximation of the mechanical power developed can be calculated by subtracting the copper losses of the armature and field winding if these losses can be calculated. The shaft power can then be calculated subtracting the mechanical losses from the mechanical power developed.

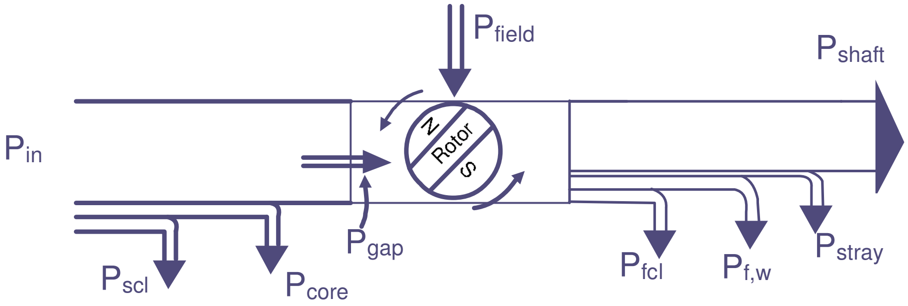


Figure 59: Power flow diagram for a synchronous motor

3 Module-3: Three phase induction motor

Overview

In this module, we first discuss about the types and constructional features of 3-phase induction motors. Principle of operation, development of equivalent circuit, derivation of torque expression are discussed in sequence. Torque-slip characteristic and its importance are also discussed. Modified Torque-slip characteristics when rotor resistance, supply voltage and supply frequency are varied are presented.

These module has 5 units covering the following aspects.

1. Types and constructional features of 3-phase induction motor.
2. Principle of operation.
3. Development of equivalent circuit.
4. Expression for electromagnetic torque
5. Torque-slip characteristic.

Module Objectives

In this module types and constructional features of 3-phase induction motor are first discussed. Principle of operation, development of equivalent circuit and expression of torque are explained and discussed. The importance of torque-slip characteristic in understanding motor operation is heighlighted.

After going through this module, students will be able to

1. explain the principle of operation of 3-phase induction motor.
2. say the range of slip for motor operation.
3. draw a typical characteristic of 3-phase induction motor and identify stable and unstable zone of operation.
4. explain why frequencies of stator and rotor electrical quantities have different values.
5. explain how operating point moves from one steady state condition to the other steady state point of operation.
6. to draw modified torque-slip characteristic when rotor resistance, supply voltage and supply frequency are varied.
7. able to explain the operation of a cage type induction motor qualitatively.

3.1 Unit-1 Types and constructional features of 3-phase induction motor

Overview

Induction motors are considered to be the work horse of industries and widely used. Unlike other motors, it works on single excitation (only stator winding is to be excited from a 3-phase source). By induction voltage is induced in the rotor coils which produce current in the rotor conductors as the rotor terminal are kept short circuited. There are two types of induction motors namely slip ring and squirrel cage types. Basic constructional features of these two types are discussed. Emphasis is to be given on relative cost and flexibility of operation of the two.

Objectives

After going through this unit, students will be able to:

1. identify the two types of induction motor by physically looking at them.
2. understand the necessity and working of slip ring and brush arrangement for accessing the rotor terminals.
3. conclude that a cage rotor in fact is equivalent to a polyphase winding.

3.1.1 Suggested material to meet the objectives

Types & Constructional features of 3-phase Induction motor

Induction motors are generally of two types namely *Slip Ring* or *Wound Rotor* type and *Squirrel Cage Type*. There is no difference so far as the *stators* of both the types are concerned. Stator houses a *balanced 3-phase distributed* winding in the slots of the laminated stator iron. Difference comes in the construction of the rotor. In case of slip ring type, rotor houses a balanced 3-phase distributed winding similar to that of the stator and the rotor terminals are brought out through slip ring and brush arrangements as shown in the figure 1. In cage induction motor a number of conductor bars are placed in rotor slots. Bar ends are kept permanently short circuited at both the ends with help conducting *end rings*. Imagine that rotor iron has been removed, then the rotor will like a *cage* as shown in figure 2, where for simplicity only 4 number of rotor bars have been shown. So for obvious reason no rotor terminals in case of cage induction motor are available.

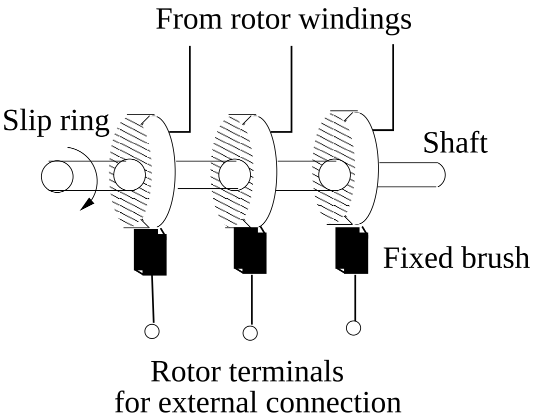


Figure 1: Slip ring & brush arrangement.

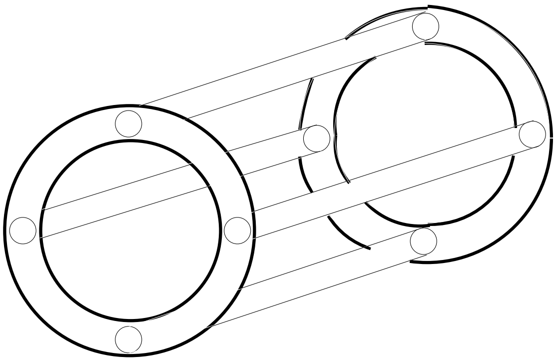


Figure 2: A squirrel cage rotor.

In the following figures 3 and 4 are shown the representation of *Slip Ring* and *Cage* type 3-phase induction motors. It may be noted that the rotor terminals of wound rotor induction motor generally be kept short circuited with the help of an external shorting switch S as shown, for cage motor, rotor bars are inherently shorted.

Cage motors are quite rugged and practically no maintenance is required on the rotor side. However regular maintenance is required as slip rings and brushes are present in wound rotor motors. The cost of a slip ring machine will be much higher than the cost of cage machine primarily because of (i) cage rotor construction is very simple and can be mass produced with unskilled labor; (ii) skilled winder is necessary to make rotor windings and (iii) extra cost is incurred for slip rings in wound motors.

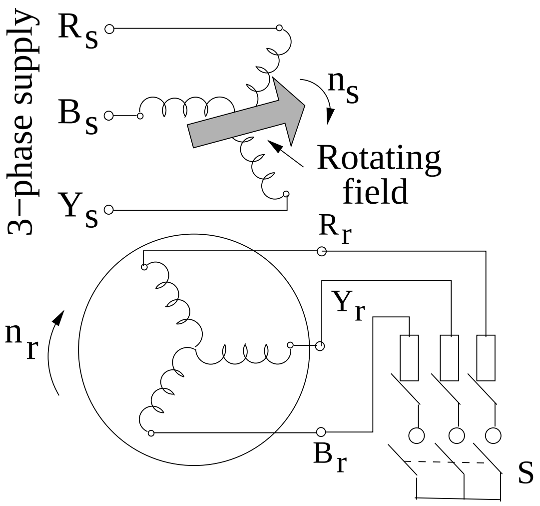


Figure 3: Slip Ring induction motor.

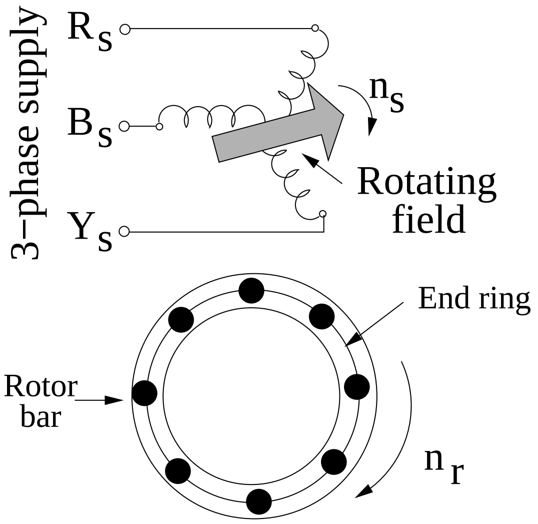


Figure 4: Squirrel Cage induction motor.

3.1.2 Unit-1 Problems

1. How will you identify the type of 3-phase induction motor by physically looking at the motor?
   Answer: In slip ring induction motor both stator and rotor terminals are brought out. There may be either three or six stator terminals. For rotor three terminals will be available. Also we know, rotor terminals are brought out through slip rings and brush arrangement. Therefore, on one side of the shaft, three numbers of slip rings along with fixed brushes can be seen by opening the end cover of the machine.

2. What is the function of *slip ring & brush* arrangement in a wound rotor induction motor?
   Answer: *slip ring & brush* arrangement is necessary to make the rotor terminals appear stationary to the user, so that we can connect meters, rheostats etc. without any problem.

3. How many rotor phases will you attribute for a cage type 3-phase, 4 pole induction motor having 20 rotor bars?
   Answer: Cage rotor will behave as $2S/p = 10$ number of phases. In other words, number of slots or rotor bars per pole pair decide the number of phases. This is because phase of the voltage will repeat after every pole pair.

4. For a slip ring induction motor, what should be done with the rotor terminals for successful operation as a motor?
   Answer: Obviously motor will not work if the rotor terminals are kept open circuited. For torque to be produced, rotor current must be present. Therefore rotor terminals either may be kept short circuited or suitable resistance may be connected across it to have improved starting torque.

3.2 Unit-2 Principle of operation

Overview

Principle of operation of 3-phase induction motor is clearly explained from fundamentals of rotating magnetic field and Faraday's laws.

Objectives

After going through this unit, students will be able to:

1. qualitatively explain the principle of operation of the motor.
2. feel the importance of slip.
3. make out that at stand still condition, the motor behaves like a transformer.
4. conclude that frequency of rotor voltage & current depend on slip.
5. that at starting slip $s = 1$ while at no load condition slip $s \approx 0$.
6. understand that the value of slip at which the motor will be operating, depends on the load torque present on the shaft.

3.2.1 Suggested material to meet the objectives

Basic principle of operation

Driving & Opposing torques

There are various kinds of rotating electrical machines such as D.C machines, Induction machines, Synchronous machines etc. and they can run either as motor or as a generator. When a generator or a motor runs at a constant speed, we can say with conviction (from Newton's laws of rotational motion) that the *driving* torque and the *opposing* torque must be numerically equal and acting in opposite directions.

Generator mode

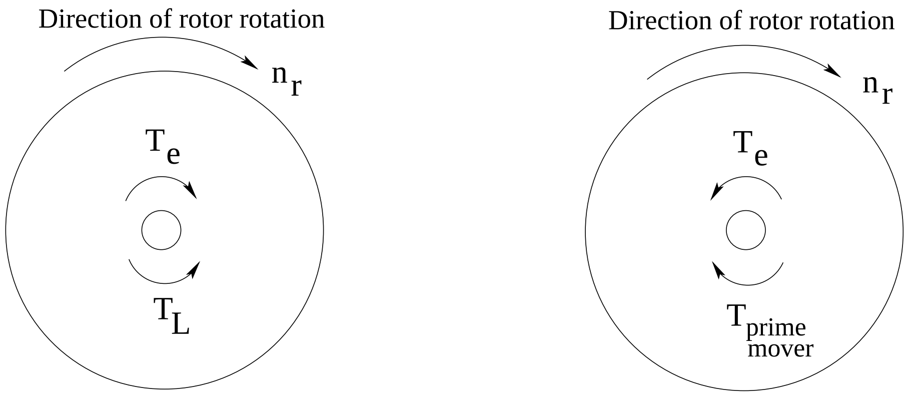
In case of generator mode, the driving torque is obtained by *prime movers.* A diesel engine or water turbine or steam turbine could be selected as prime movers. In laboratory environment, motors are used as prime movers. The direction of rotation of the generator is same as the direction of the prime mover torque. A loaded electrical rotating machine always produces electromagnetic torque T_e, due to the interaction of stator field and armature current. T_e together with small frictional torque is the opposing torque in generator mode. This opposing torque is called the *load* torque, T_L. If one wants draw more electrical power out of the generator, T_e (hence T_L) increases due to more armature current. Therefore, prime mover torque must increase to balance T_L for steady speed operation with more fuel intake.

Motor mode

In case of motor mode, the driving torque is the electromagnetic torque, T_e and direction of rotation will be along the direction of T_e. Here the opposing torque will be due to mechanical load (such as pumps, lift, crane,blower etc.) put on the shaft and small frictional torque. In this case also the opposing torque is called the load torque T_L. For steady speed operation, $T_e = T_L$ numerically and acts in opposite direction. To summarize, remember:

- if it is acting as a motor, electromagnetic torque T_e acts along the direction of the rotor rotation and the load torque T_L acts in the opposite direction of rotation as shown in the figure 5 (a). If $T_e = T_L$ motor runs steadily at constant speed. During transient operation, if $T_e > T_L$ motor will accelerate and if $T_e < T_L$ motor will decelerate.

- On the other hand, if the machine is acting as a generator, the prime mover torque T_{pm} acts along the direction of rotation while the electromagnetic torque, T_e acts in the opposite direction of rotation as shown in figure 5 (b). Here also during transient operation if $T_{pm} > T_L$, the generator will accelerate and if $T_{pm} < T_L$, the generator will decelerate.



(a) Motor mode (b) Generator mode

Figure 5: Direction of torques in rotating machines.

Let us first consider a slip ring motor to understand the principle of operation. Initially suppose the machine is stationary (i.e., rotor speed $n_r = 0$) and the rotor terminals R_r, Y_r and B_r are opened. If now 3-phase, f Hz supply is given to the stator terminals, a rotating magnetic field results moving with a speed n_s with respect to the stator or a stationary observer. This field will induce voltages in the rotor coils of frequency f Hz. Rotor conductors, although placed in a magnetic field, can not experience any force or torque as they don't carry any current by virtue of the fact that they have been left open circuited. As same flux links both the the stator and the rotor coils, the ratio of the induced voltages in stator and rotor phase coils will be in the ratio of their effective number of turns like a transformer.

If now the rotor terminals are shorted by say a shorting switch S, rotor conductors will now carry currents and experience torque. The rotor starts rotating in the same direction as that the direction of the stator field. Why? This is left to you to ascertain this. To give some hints you may apply, *left hand rule* or *Lenz's law* with which you are well familiar with. Note that when the machine is about to start, induction motor behaves in the same way as a transformer whose secondary is shorted. The frequency of rotor current is still f.

The scenario changes once rotor starts accelerating. Suppose, rotor has attained a speed n_r at any time t during the course acceleration. The magnitude of the rotor induced voltage and its frequency must have decreased, as anybody sitting on the rotor conductor will see the rotating field move past him at a speed of $n_s - n_r$ and not n_s at the time of starting. Consequently, the current in the short circuited secondary too will decrease. Nonetheless, electromagnetic torque still will be produced thereby accelerating the rotor further. Now the question is where the motor will finally settle down? To understand this let us assume a rather ideal situation where no friction exist on the bearing. So long $n_r < n_s$, we have seen that the motor speed will go on increasing and a time will come when $n_r = n_s$, making relative speed between the rotating field and the rotor zero. Hence induced voltage and rotor currents vanish. Electromagnetic torque, T_e also becomes zero. It is however not surprising at all, because we know from Newton,s laws of motion that *to rotate a body at some constant angular speed in an absolutely frictionless environment, no external torque is necessary.* Therefore the motor will finally settle down at $n_r = n_s$. A motor is purchased and installed not for running it on no load but to supply some mechanical load such as pumps, lift and many others. Essentially, a mechanical load imposes on the shaft of the motor a load torque, T_L which acts in the opposite direction of rotation. Therefore, for a mechanically loaded motor, the final operating speed of the motor will be once again such that electromagnetic torque developed will be equal to the load torque i,e.,$T_e = T_L$. Since $T_L > T_{fric}$ rotor speed of a loaded induction motor, $n_r < n_{ro} < n_s$. Thus as the load torque increases, the speed of the motor falls. For a typical well designed induction motor, the variation of speed from no load to the full load condition is very small, of the order of 3 to 5%.

Slip speed & slip

From the above discussion, it is expected, the difference of the synchronous speed and the rotor speed i.e., $n_s - n_r$ is going to play a crucial role in deciding torque developed. This difference of speed is called the *slip speed*. Remembering that n_s is constant and n_r depends upon load present on the shaft another term called *slip* is defined as $s = (n_s - n_r)/n_s$. This is nothing but the slip speed expressed as per unit of synchronous speed. Slip is often expressed in percentage as $s = (n_s - n_r) \times 100/n_s$ %. In the next section we shall see that the induced voltage in the rotor and its frequency can be nicely expressed in terms of slip. Knowledge of slip indirectly give us rotor speed as $n_r = (1 - s)n_s$.

3.2.2 Unit-2 Problems

1. Why does the rotor moves in the same direction as that of the stator rotating field?
   Answer: We know from Lenz's law, that induced voltage in a coil will act in such a way that it will try to oppose the very cause for which the induced emf is due. The cause of induced voltage in the rotor coils is due to the relative speed of the stator field and the rotor coils. Thus rotor will start rotating in such a direction so as to reduce the relative speed. This can happen if the rotor moves in the same direction as that of the stator field.

2. What does the rotor do when

   (a) electromagnetic torque is greater than the load torque in a motor.
   (b) electromagnetic torque is less than the load torque in a motor.
   (c) electromagnetic torque is less than the prime mover torque in a generator.
   (d) electromagnetic torque is more than the prime mover torque in a generator.

3. With the help of a diagram show the direction of rotation, direction of electromagnetic torque and load torque for motor.

4. With the help of a diagram show the direction of rotation, direction of electromagnetic torque and load torque for generator.

5. How many slots will be necessary, to make a 3-phase distributed winding which will produce 6-pole.
   Solution
   We know that to produce 2-pole we require one coil hence 2 slots. Therefore we shall require a minimum of 3 coils (6 slots) to produce 6-pole per phase. Each phase has to produce 6 pole individually. Therefore total number of slots required will be $3 \times 6 = 18$.

3.3 Unit-3 Development of equivalent circuit

Overview

In this unit per phase equivalent circuit of a 3-phase induction motor is developed. It is similar to that of a static transformer but with an important difference. The important difference is due to the fact that power output of the motor is mechanical in nature and this is to be represented in the form of a speed or slip dependent resistance. Usefulness of the equivalent circuit in predicting the performance of the motor is highlighted.

Objectives

After going through this unit, students will be able to:

1. draw the equivalent circuit of the motor when running at a slip and identify various parameters.
2. understand how power flows from input to the mechanical load.
3. enumerate various losses that take place in the motor.
4. prove that Air gap power : Rotor copper loss : Gross mechanical power is $1 : s : (1 - s)$.
5. draw approximate equivalent circuit neglecting stator impedance.

3.3.1 Suggested material to meet the objectives

Analysis & equivalent circuit

Let us assume that the induction machine is operating as motor and running with a slip s. The air gap flux density distribution is assumed to be sinusoidal fundamental component, other harmonic components being neglected.

$$\begin{aligned}
\text{Flux per pole, } \phi &= \frac{4}{p}B_{max}lr \\
\text{where, } B_{max} &= \text{Maximum value of flux density} \\
l &= \text{Length of the motor.} \\
r &= \text{Radius of the rotor.} \\
p &= \text{Number of pole of the machine.} \\
N_1 &= \text{Actual stator number of turns per phase.} \\
k_{w1} &= \text{Winding factor of stator.} \\
k_{w1}N_1 &= \text{Effective stator number of turns per phase.} \\
N_2 &= \text{Actual rotor number of turns per phase.} \\
k_{w2} &= \text{Winding factor of rotor.} \\
k_{w2}N_2 &= \text{Effective rotor number of turns per phase.}
\end{aligned}$$

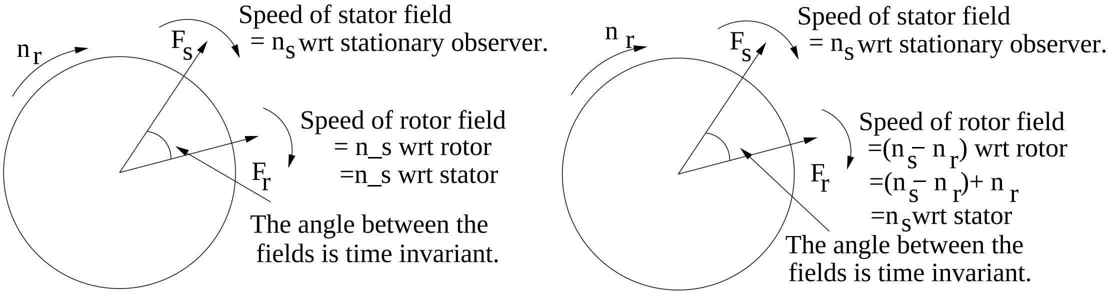
Also let us define the following:

$$\begin{aligned}
\text{Stator induced voltage per phase, } E_2 &= \sqrt{2}\pi f\phi k_{w1}N_1 \\
\text{Rotor induced voltage per phase at } \textit{stand still}\text{condition, } E_{2S} &= \sqrt{2}\pi f\phi k_{w2}N_2 \\
\text{Rotor induced voltage per phase at a slip } s\text{, } E_2 &= \sqrt{2}\pi sf\phi k_{w2}N_2 \\
&= sE_{2s}
\end{aligned}$$

Although the stator electrical quantities have frequency f, it is interesting to note that the frequency of rotor voltage and current is sf. This can easily be shown as follows:

$$\begin{aligned}
\text{Mechanical relative speed between stator field \& rotor conductors} &= (n_s - n_r) \\
\text{Frequency of rotor voltage \& current } f_r &= \frac{p}{2}(n_s - n_r) \\
&= \frac{p}{2} s n_s \\
&= sf
\end{aligned}$$

If supply frequency is 50Hz and full load operating slip is about 5%, the rotor frequency will be only 2.5Hz. At stand still condition, s being 1, frequency of stator & rotor currents are same at f and induction motor behaves just like a short circuited transformer. Also the flux level (i.e., flux per pole) will remain practically constant from no load to full load condition and is decided by the supply voltage and supply frequency. The no load condition in case of an induction motor essentially means $s = 0$ when rotor induced voltage and current are zero. Current drawn from the supply will be no load current I_0 comprising of magnetizing current I_m and the core loss component I_{cl} similar to that of a practical transformer. Let us see physically what happens when the induction motor is at stand still condition i.e., at slip $s = 1$. Frequency of rotor current is now equal to supply frequency f. So the three phase winding of the rotor, carries balanced three phase current. Hence rotor too, will produce a rotating magnetic field with a speed of rotation $\frac{2f}{p}(= n_s)$ with respect to rotor structure. A stationary observer sees two rotating field namely stator field and rotor field moving in the same direction with same speed. So angle between the fields is time invariant. the situation is depicted in the figure 6(a). This is another way of explaining the fact that a 3-phase induction motor has a starting torque.



(a) Situation at slip s=1

(b) Situation at slip s

Figure 6: The angle between stator & rotor field is always time invariant.

Let us now suppose that the machine is running with some slip at a speed of n_r in the clockwise direction. Then,

$$\begin{aligned}
\text{Frequency of stator current} &= f \\
\text{Speed of the stator field wrt stationary observer} &= n_s \ (= \frac{2f}{p}) \\
\text{Frequency of rotor current} &= f_r \ (= \frac{p(n_s - n_r)}{2}) \\
\text{Speed of the rotor field wrt rotor} &= (n_s - n_r) \\
\text{Speed of the rotor field wrt stator} &= (n_s - n_r) + n_r \\
&= n_s
\end{aligned}$$

In this case also we find that the angle between the stator & rotor field is time invariant. Any body sitting on the stator, as such can not distinguish whether the situation corresponds to standstill or arbitrary running condition. Because in both the cases, stator sees that a rotating magnetic field has been produced by the rotor moving with a speed n_s.

Whenever there will be current in the rotor circuit, to keep the flux level constant in the air gap of the machine, stator coils will draw additional (reflected current) from the supply to counter the field of rotor. At what frequency, will this additional current will be drawn? This gets decided by the fact that it has to counter a rotor rotating field moving at a speed of n_s wrt to stator. By drawing only 50Hz current from the supply, such a feat can be achieved. In other words, although frequency of rotor current changes with the degree of loading, the stator current frequency remains unchanged at the supply frequency 50Hz. We thus conclude that while during standstill condition induction machine strictly behaves like a transformer, during running condition it is not. During running condition there is reflected current no doubt but frequency of stator and rotor winding currents are different.

Equivalent circuit

The discussion on development of equivalent circuit Will be done in two levels. In the first level stand still condition is considered and in the second level running with an arbitrary slip is considered. So far as the stand still condition is concerned, the per phase equivalent circuit will be exactly similar to that of a transformer. Figures 7 and 8 shows the per phase equivalent circuit of the induction motor at $s = 1$. In similar line with the concept of transformer the per phase equivalent circuit parameters are defined as follows:

$$\begin{aligned}
\text{Stator to rotor phase turns ratio, } a &= \frac{k_{w1}N_1}{k_{w2}N_2} \\
\text{Stator resistance} &= r_1 \\
\text{Stator leakage reactance} &= x_1 \\
\text{Magnetizing reactance} &= x_m \\
\text{Resistance representing core loss} &= R_{cl} \\
\text{Rotor resistance} &= r_2 \\
\text{Rotor leakage reactance at } \textit{standstill} \text{ condition} &= x_2 \\
\text{Rotor resistance referred to stator, } r_2' &= a^2 r_2 \\
\text{Rotor leakage reactance referred to stator, } x_2' &= a^2 x_2
\end{aligned}$$

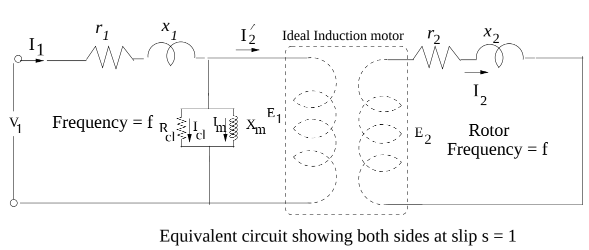


Figure 7: Equivalent circuit showing both sides.

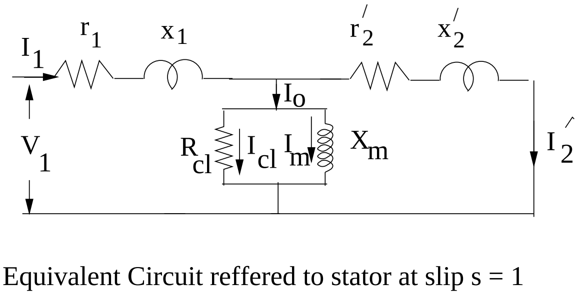


Figure 8: Equivalent circuit ref. to stator.

Now let us consider the actual rotor circuit shown in figure 9, when the motor runs with a slip s. here induced voltage in the rotor is sE_2 and the rotor leakage impedance is $r_2 + jsx_2$ where x_2

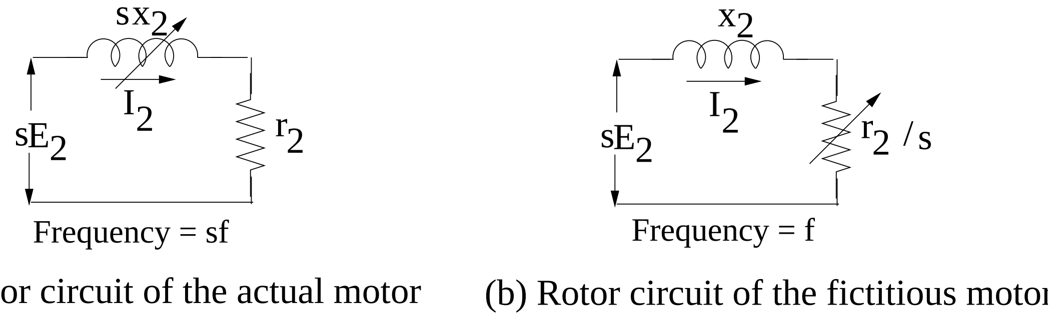
is the rotor leakage reactance at standstill condition. The current in the rotor circuit is

$$\begin{aligned}
\text{Actual rotor current, } I_2 &= \frac{sE_2}{\sqrt{r_2^2+(sx_2)^2}} \\
\text{Frequency of rotor current, } f_r &= sf \\
\text{Corresponding reflected stator current, } I_2' &= \frac{k_{w2}N_2}{k_{w1}N_1} I_2 \\
\text{Stator current frequency} &= f
\end{aligned}$$

The magnitude of this reflected current, we should note, depends on the magnitude of the rotor current and the ratio of effective number of turns and not on frequency of the rotor current. This piece of fact can be applied effectively for obtaining equivalent circuit of the induction motor operating at slip s. In figure 9(a) is shown the actual rotor equivalent circuit operating at sf Hz. Certainly the stator and the rotor circuits can not be connected simply by changing the level of voltage to a common value as the frequency of the two circuits are different. Had the frequencies been same there would not have been any problem as already seen for the standstill condition. Therefore, before attempting any interconnection of the two sides like transformer, we have to somehow transform the rotor circuit in such away that its rotor current remains same as the actual value but frequency becomes f.

Let us imagine a fictitious induction motor whose turns ratio is same as that of the original machine and whose *standstill leakage rotor impedance* is $(\frac{r_2}{s}+jx_2)$ as shown in figure 9(b). For this machine at standstill condition the rotor current is given by

$$\begin{aligned}
\text{standstill rotor current of the fictitious motor } I_2 &= \frac{E_2}{\sqrt{(\frac{r_2}{s})^2+(x_2)^2}} \\
&= \frac{sE_2}{\sqrt{r_2^2+(sx_2)^2}} \\
&= \text{same as the rotor current of the actual machine ru}
\end{aligned}$$



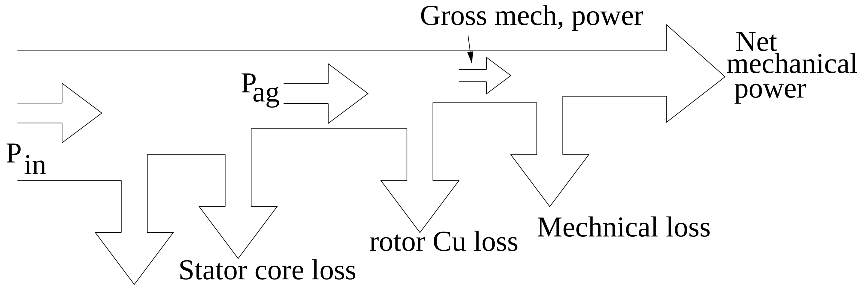
(a) Rotor circuit of the actual motor (b) Rotor circuit of the fictitious motoı

Figure 9: Rotor circuit.

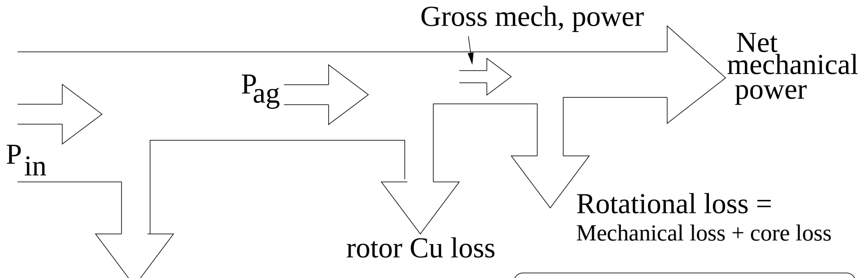
The stator will react in the same way (i.e., drawing same reflected current at the same power factor) for both the actual and the fictitious machine. So the equivalent circuit of the fictitious machine can be easily drawn and we realize that this also represents the equivalent circuit of the actual motor running with slip s. The exact and the approximate per phase equivalent circuits of the induction motor running with slip s are shown in figure 11(a) and (b).

3.3.2 Power flow diagram

Power flow diagram, shown in figure 10 gives us a pictorial view of the power distribution inside the machine, starting from the *electrical power input* finally to the *Net mechanical power output*. In the stator, the copper loss in the winding and the core loss occurs. From the total input if we subtract these two losses we get the power which crosses the air gap and enters into the rotor. This power is called the *Air Gap Power*. In the rotor rotor copper loss takes place and if this is subtracted from the air gap power we get the gross mechanical power developed. The net mechanical power will be obtained if mechanical loss (frictional & windage losses) is subtracted from the gross mechanical power.



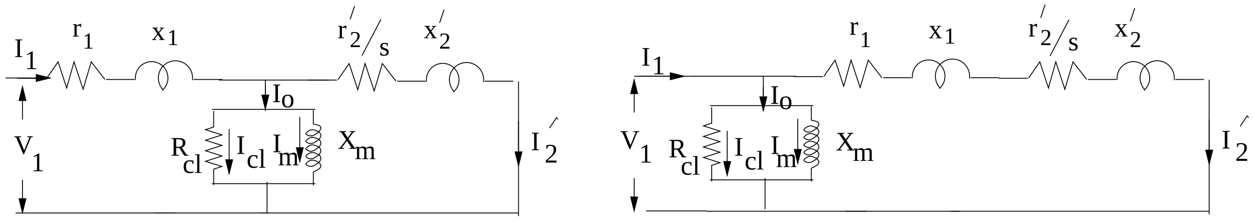
Exact Power Flow Diagram



Approximate Power Flow Diagram

Figure 10: Power flow diagram.

Bringing the parallel branch of impedance right across the supply may not be that accurate as in case of transformer but one can get a fairly good idea of the performance of the motor for any operating slip rather quickly. In fact to simplify matters further, the stator impedance is often neglected for getting handy and simplified expressions for torque, power etc. This is somewhat justified by the fact that the normal operating slip of induction motor will be small of the order of about 5%, making $\frac{r_2'}{s}$ hence rotor impedance z_2' quite large compared to stator impedance.



(a) Exact equivalent Circuit reffered to stator at slip s

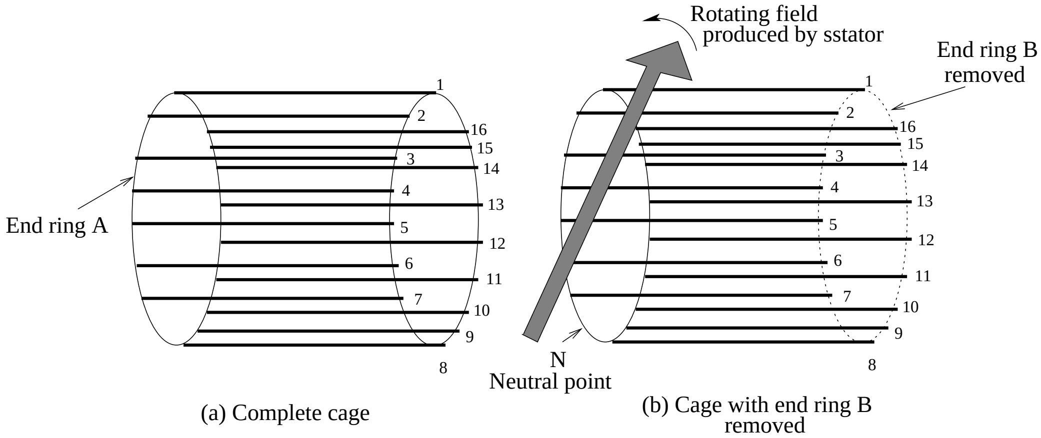
(b) Approx. equivalent Circuit reffered to stator at slip s

Figure 11: Equivalent circuit for slip s.

3.3.3 More on cage rotor induction motor

Explanation of principle of operation and development of equivalent circuit of induction motor have been carried out assuming the motor to be wound rotor type. Question arises whether can we apply these to cage type induction motor as well? The answer to this is yes. We try to justify this qualitatively in the following paragraph.

We know a rotating magnetic field can be produced by a balanced 2-phase or 3-phase or 6-phase or (in general) poly phase winding. The implication of this is far reaching in the sense that we can, if we desire make a induction motor whose stator houses a balanced 3-phase winding and the rotor houses a balanced 2-phase winding. This machine will work all right provided the stator and rotor number of poles are equal. As we know, production of steady torque is due to the interaction of stator and rotor poles - it does not matter whether these poles have been produced by a balanced 2-phase or 3-phase or 6-phase or (in general) poly phase winding. With this understanding we are in a position to explain how a cage motor works.

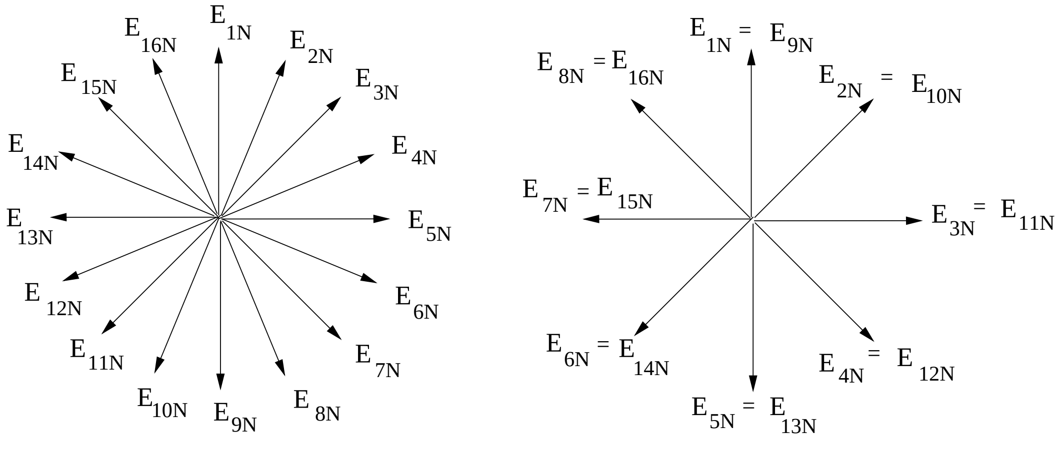


(a) Complete cage

(b) Cage with end ring B removed

Figure 12: Cage rotor

In figure 12(a), a cage rotor with two end rings A and B short circuiting all the 16 bars at both the ends. Let us imagine that end ring B has been removed so that rotor bar become open circuited with terminals 1, 2, … 16 and the other ends of the bars remain short circuited through end ring A and let that common point be marked as N (neutral). Suppose rotating field produced by stator moves in the anti clockwise direction as shown. Therefore in each of the bar voltage ($b\,l\,v$) will be induced. Magnitude of the RMS voltage induced in each bar will be same. However, voltage in bar-2 will lag the voltage in bar-1 by slot angle. Let the stator produces 2-pole. Therefore voltage phasors of all the 16 bars can be drawn as $E_{1N}, E_{2N}, \dots E_{16N}$ as shown in figure 13(a).



(a) Bar voltages for 2 poles

(b) Bar voltages for 4 poles

Figure 13: Phasor for bar voltages

So we now have a balanced 16-phase voltage induced in the rotor circuit which has been kept opened (as end ring-B has been removed). Let us insert the end ring-B at the other end (like short circuiting the terminals of a slip ring motor). Naturally, balanced 16 phase current will exist in the bars and rotor will be able to produce its own rotating field (since any poly phase distributed current produces a rotating field). Stator does not really know whether the rotor field has been produced by a 3-phase or in general by a poly phase current distribution in the rotor.

Another interesting point with cage rotor is its capability to adjust automatically with stator number of poles. Imagine that the stator winding (which produced 2-pole) of the above machine is replaced by a 3-phase winding which produce 4-pole. However, the same cage rotor with 16 bars is used. It will be found, that the motor will once again run successfully. In this case, the bars which are 2-pole pitch apart will have induced co-phasal voltages. Voltages in bars 1and9, 2and10 . . . 8and16 will be in same phase as shown in figure 13(b). Thus over 360° mechanical angle we shall have 2 cycles of identical current distribution in space ensuring 4-pole will now be produced by the rotor. Conclusion is, if number of stator pole is changed, rotor will adjust itself to produce same number of stator poles. In general, if S is the total number of bars and p is the number of pole then we can attribute $2S/p$ number of phases to the cage rotor.

3.3.4 Unit-3 Problems

1. Why does the rotor moves in the same direction as that of the stator rotating field?
   Answer: We know from Lenz's law, that induced voltage in a coil will act in such a way that it will try to oppose the very cause for which the induced emf is due. The cause of induced voltage in the rotor coils is due to the relative speed of the stator field and the rotor coils. Thus rotor will start rotating in such a direction so as to reduce the relative speed. This can happen if the rotor moves in the same direction as that of the stator field.

2. Calculate the value of the slip s for a 3-phase, 6 pole, 50 Hz induction motor running at 970 rpm.
   Solution:

$$\begin{aligned}\text{Synchronous speed, } n_s &= 120 \times f/p = 120 \times 50/6 = 1000 \text{ rpm}\\ \text{rotor speed, } n_r &= 970 \text{ rpm}\\ \text{slip, } s &= (n_s - n_r)/n_s = (1000 - 970)/1000 = 0.03 = 3\,\%\end{aligned}$$

3. A 3-phase, 4 pole induction motor is found to run at 1728 rpm at a slip of 4%. Calculate the frequency of the supply.
   Solution:

$$\begin{aligned}\text{synchronous speed, } n_s &= n_r/(1 - s) = 1728/(1 - 0.04) = 1838.3 \text{ rpm}\\ \text{or, } 120 \times f/4 &= 1838.3\\ \text{or, } f &= 1838.3 \times 4/120 = 61.28\,\text{Hz}\end{aligned}$$

4. A 3-phase, 50 Hz, 4 pole induction motor is found to run at 1440 rpm. What are the frequencies of stator current and rotor current?
   Solution:

$$\begin{aligned}\text{synchronous speed, } n_s &= 120 \times f/4 = 120 \times 50/4 = 1500 \text{ rpm}\\ \text{slip, } s &= (n_s - n_r)/n_s = (1500 - 1440)/1500 = 0.04 = 4\,\%\\ \text{Frequency of stator current, } f &= 50\,\text{Hz}\\ \text{Frequency of rotor current, } f_r &= s \times f = 0.04 \times 50 = 2\,\text{Hz}\end{aligned}$$

5. A 3-phase, 50 Hz, 6 pole induction motor is found to run at 950 rpm. Calculate the
   (i) speed of the stator rotating field with respect to stator.
   (ii) speed of the stator rotating field with respect to rotor.
   (iii) speed of the rotor rotating field with respect to rotor.
   (iv) speed of the rotor rotating field with respect to stator.

   Solution

$$\begin{aligned}
\text{synchronous speed, } n_s &= 120 \times f/6 = 120 \times 50/6 = 1000\,\text{rpm} \\
\text{speed of the stator field wrt stator } n_s &= 1000\,\text{rpm} \\
\text{speed of the stator field wrt rotor } n_s - n_r &= 1000 - 950 = 50\,\text{rpm} \\
\text{Slip of the motor } s &= (1000 - 950)/1000 = .05 \\
\text{Frequency of rotor current, } f_r &= s \times f = 2.5\,\text{Hz} \\
\text{speed of the rotor field wrt rotor} &= 120 \times f_r/6 = 50\,\text{rpm} \\
\text{speed of the rotor field wrt stator} &= 950 + 50 = 1000\,\text{rpm}
\end{aligned}$$

6. If n_s is the synchronous speed of an induction motor, prove that the relative speed of the fields produced by stator and rotor is zero when the motor runs at any arbitrary speed of n_r.
   Solution

$$\begin{aligned}
\text{speed of the stator field wrt stator} &= n_s \\
\text{speed of the rotor field wrt rotor} &= n_s - n_r \\
\text{speed of the rotor field wrt stator} &= n_s - n_r + n_r = n_s
\end{aligned}$$

   Thus the relative speed between stator and rotor fields zero at all n_r.

7. A 3-phase induction motor is found to run at 5% slip with per phase induced voltage of 10 V. What will be the per phase rotor induced voltage
   (a) at stand still condition?
   (b) when rotor runs at 3% slip? Let the rotor induced voltage per phase at stand still condition be E_2.

   Solution

$$\begin{aligned}
\therefore \text{ induced voltage at slip } s &= s \times E_2 \\
\text{induced voltage at slip } .05 &= .05 \times E_2 = 10\,\text{V} \\
\text{induced voltage at stand still condition } E_2 &= 10/.05 = 200\,\text{V} \\
\therefore \text{ induced voltage at slip } .03 &= .03 \times E_2 = 6\,\text{V}
\end{aligned}$$

8. Sketch the power flow diagram of a 3-phase induction motor showing various losses that take place in the motor.

9. How air gap power, rotor copper loss and gross mechanical power are related when the motor runs at slip s.
   Solution

$$\text{Air gap power} : \text{Rotor Cu loss} : \text{Gross mech power} = 1 : s : (1-s)$$

10. Where does core loss occur in induction motor? Why core loss in rotor iron can be neglected?
    Solution
    We know the with respect to stator iron air gap field rotates at n_s and with respect to rotor iron air gap field rotates at $n_s - n_r$. Now the slip speed is quite small. Hence the core loss primarily occurs in stator iron and core loss in rotor iron can be practically neglected.

11. What is rotational loss? What are the different component of this loss.
    Answer: Rotational loss is sum of core loss and friction & windage loss. Since variation of speed from no load to full load is quite small friction & windage loss can be considered to be constant. Also note, flux level of the machine practically remains constant from no load to full load. So rotational loss is assumed to be constant for all practical purposes.

12. What parameters in the equivalent circuit are involved in determining (i) air gap power (ii) gross mechanical power.
    Answer: For air gap power it is r_2'/s and for mechanical power it is $(1-s)r_2'/s$.

13. The input power to a 3-phase induction motor is 7.5 kW when running at a slip s. At this condition, stator copper loss, rotor copper loss and rotational losses are respectively 200 W, 300 W and 500 W. Calculate the air gap power, slip, net mechanical power output and efficiency of the motor.
    Solution

$$\begin{aligned}
\text{Input power}, P_{in} &= 7.5\,\text{kW} \\
\text{Air gap power}, P_{ag} &= P_{in} - \text{Stator loss} = 7.5 - 0.2 = 7.3\,\text{kW} \\
\text{Rotor Cu loss}, P_{\text{rotculoss}} &= s \times P_{ag} = s \times 7.3 = 0.3 \\
\therefore\ \text{slip}, s &= 0.3/7.3 = 0.041 \\
P_{\text{grossmech}} &= (1-s)P_{ag} = (1-0.401) \times 7.3 = 7.008\,\text{kW} \\
P_{\text{netmech}} &= P_{\text{grossmech}} - \text{ rotational loss} = 7.008 - 0.5 = 6.508\,\text{kW} \\
\text{Efficiency of the motor}, \eta &= P_{\text{netmech}}/P_{in} = (6.508/7.50) \times 100 = 86.7\%
\end{aligned}$$

14. Gross mechanical power developed by a 6 pole, 50 Hz, 3-phase induction motor is 8 kW while running at 960 rpm. Calculate the rotor copper loss.
    Solution

$$\begin{aligned}
\text{synchronous speed}, n_s &= 120 \times f/6 = 120 \times 50/6 = 1000\,\text{rpm} \\
\text{rotor speed}, n_r &= 950\,\text{rpm} \\
\text{slip}, s &= (1000 - 950)/1000 = 0.05 \\
\text{Air gap power}, P_{ag} &= P_{\text{grossmech}}/(1-s) = 8/(1-.05) = 8.42\,\text{kW} \\
\text{Rotor Cu loss}, P_{\text{rotculoss}} &= s \times P_{ag} 0.05 \times 8.42 = 0.421\,\text{kW} = 421\,\text{W}
\end{aligned}$$

3.4 Unit-4 Torque Equation & Torque-slip Characteristic

Overview

In this unit expression for the electromagnetic torque is derived in terms of the equivalent circuit parameters. Typical torque versus slip characteristic is drawn when slip s, varies from 1 to 0.

Objectives

After going through this unit, students will be able to:

1. identify the parameter of the equivalent circuit which represent gross mechanical power developed.
2. conclude that torque developed is also given by air gap power divided by synchronous speed.
3. express torque in synchronous watt (which is nothing but the air gap power in watts) as the synchronous speed is constant.
4. calculate the torque at any slip.
5. sketch typical torque versus slip characteristic

3.4.1 Suggested material to meet the objectives

Torque expression neglecting stator impedance

To bring out some important characteristics of induction motor, often a very simplified model of induction motor equivalent circuit is taken up by neglecting stator impedance and the parallel impedance representing the no load current. In this case the power input, indeed becomes the *air gap power* and the equivalent circuit looks like (14):

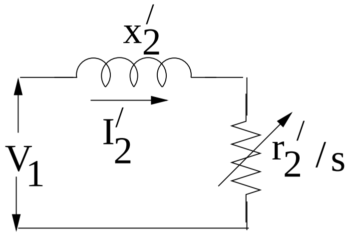


Figure 14: Equivalent circuit neglecting stator impedance ref. to stator for slip s.

$$\begin{aligned}
\text{Current drawn from supply } I_2' &= \frac{V_1}{\sqrt{(\frac{r_2'}{s})^2 + (x_2')^2}} \\
\text{Air gap power } P_{ag} &= 3{I_2'}^2\frac{r_2'}{s} \\
\text{Rotor copper loss} &= 3{I_2'}^2 r_2' = sP_{ag} \\
\text{gross mechanical power developed} &= P_{ag} - sP_{ag} = (1-s)P_{ag} \\
\text{Gross Torque developed } T_e &= \frac{1}{2\pi n_r}(1-s)P_{ag} \\
&= \frac{1}{2\pi(1-s)n_s}(1-s)P_{ag} = \frac{P_{ag}}{2\pi n_s} = \frac{1}{2\pi n_s}3{I_2'}^2\frac{r_2'}{s}
\end{aligned}$$

The expression of torque can now be expressed in terms of equivalent circuit parameters by putting $I_2' = \frac{V_1}{\sqrt{(\frac{r_2'}{s})^2+(x_2')^2}}$ in the above torque equation.

$$
\begin{aligned}
T_e &= \frac{3}{2\pi n_s}\left[\frac{V_1}{\sqrt{(\frac{r_2'}{s})^2+(x_2')^2}}\right]^2 \frac{r_2'}{s} \\
&= \frac{3}{2\pi n_s}\frac{V_1^2\frac{r_2'}{s}}{\left[(\frac{r_2'}{s})^2+x_2'^2\right]} \\
T_e &= \frac{3V_1^2}{2\pi n_s x_2'}\frac{s\frac{r_2'}{x_2'}}{\left[s^2+(\frac{r_2'}{x_2'})^2\right]}
\end{aligned}
$$

One can further simplify the above equation by defining $\alpha = \frac{r_2'}{x_2'}$ and $K = \frac{3V_1^2}{2\pi n_s x_2'}$ to get the following handy and easy to remember torque formula.

$$T_e = K\frac{s\alpha}{s^2+\alpha^2}$$

The value of slip may vary from 0 to 1 for motoring mode - $s = 1$ corresponds to stationary rotor ($n_r = 0,$) and $s = 0$ corresponds to rotor speed to be synchronous ($n_r = n_s$). One can calculate torque values for different slip values and plot them to get the typical torque vs slip characteristic of the3-phase induction motor as shown in figure 15 The value of maximum torque T_{max} and the corresponding slip at which it occurs can be easily calculated either from the equation $\frac{dT_e}{ds} = 0$ or more elegantly by applying maximum power transfer theorem to the equivalent circuit. We know air gap power ($3I_2'^2\frac{r_2'}{s}$ in figure 14) is in fact a measure of torque in synchronous watt. Naturally the power in$\frac{r_2'}{s}$ will be maximum when $\frac{r_2'}{s} = x_2'$ or the value of slip at which maximum torque occurs is $\frac{r_2'}{x_2'} = \alpha$. After putting $s = \alpha$ in the torque equation, we get $T_{max} = \frac{K}{2}$. With stator impedance neglected, follow points should be highlighted about torque equation.

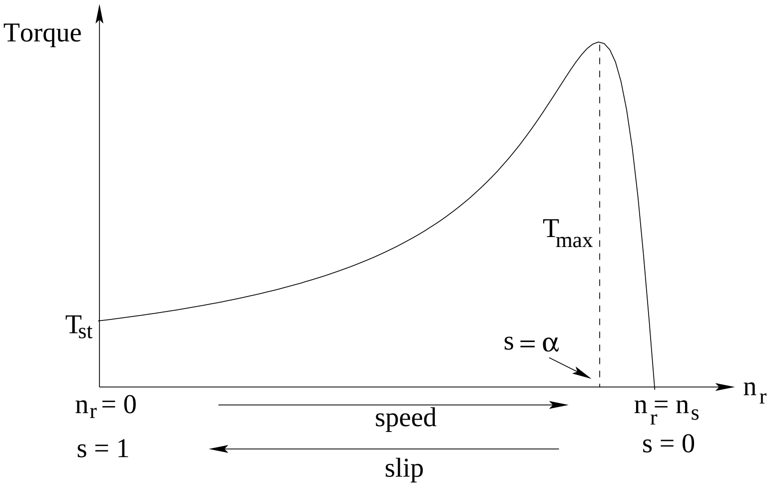


Figure 15: Torque vs slip characteristic.

- For any given slip, torque can be found from the equation $T_e = K\frac{s\alpha}{s^2+\alpha^2}$, where $K = \frac{3V_1^2}{2\pi n_s x_2'}$ is a constant.

- At starting $s = 1$, And starting torque is $T_{st} = K\frac{\alpha}{1+\alpha^2}$.
- The slip at which maximum torque occurs is α and it is directly proportional to rotor resistance.
- The value of maximum torque is $T_{max} = \frac{K}{2}$. Obviously, it is also independent of rotor resistance.
- At synchronous speed ($n_r = n_s$), slip is zero and torque developed is also zero.

More accurate expression of the torque can be found out by incorporating stator leakage impedance $r_1 + jx_1$ in the equivalent circuit as shown in figure 16

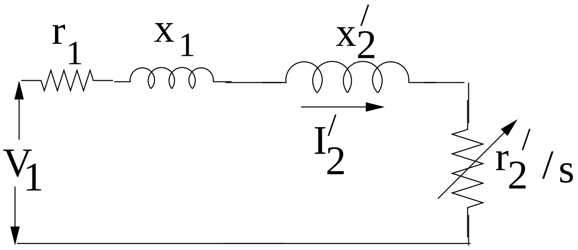


Figure 16: Equivalent circuit with both stator and rotor impedances.

So current drawn from the supply is given by

$$I_2' = \frac{V_1}{\sqrt{\left[r_1 + (\frac{r_2'}{s})\right]^2 + (x_1 + x_2')^2}}$$

Therefore, torque developed can be found out in the same fashion i.e., first calculate air gap power and then divide it by $2\pi n_s$ as shown below.

$$\begin{aligned}
\text{Air gap power } P_{ag} &= 3I_2'^2\frac{r_2'}{s} \\
T_e &= \frac{1}{2\pi n_s}3I_2'^2\frac{r_2'}{s} \\
T_e &= \frac{3}{2\pi n_s}\frac{V_1^2\frac{r_2'}{s}}{\left(r_1 + \frac{r_2'}{s}\right)^2 + (x_1 + x_2')^2}
\end{aligned}$$

Once again by applying maximum transfer theorem, the slip at which maximum torque occurs can be found out as shown below.

$$\begin{aligned}
\frac{r_2'}{s} &= \sqrt{r_1^2 + (x_1 + x_2')^2} \\
\therefore \text{the slip at which maximum torque} &= \frac{r_2'}{\sqrt{r_1^2 + (x_1 + x_2')^2}}
\end{aligned}$$

By putting the above value of slip in the torque expression and simplifying, maximum value of torque becomes

$$T_{max} = \frac{3V_1^2}{4\pi n_s}\frac{1}{\left[r_1 + \sqrt{r_1^2 + (x_1 + x_2')^2}\right]}$$

Once again the following points may be noted:

- maximum value of the torque is independent of the rotor resistance.
- maximum value of the torque is proportional to the square of the supply voltage.
- slip at which maximum torque occurs depends on rotor resistance.

How the steady operating point is obtained after the motor is started

We have touched upon this topic, while explaining the principle of operation. Let us now take help of the torque-slip characteristic to get more insight into it. Consider that a 3-phase induction motor is being started from rest under no load condition. The machine has to over come the constant frictional torque T_{fric}. The torque-slip curve along with constant T_{fric} are shown in figure 17. For the time being forget about the T_L line (as if it is not there). We see that at the time of starting $T_e = T_{st} > T_{fric}$, hence the motor will accelerate and speed will increase. The operating point moves along path ABCDEFP$_o$. At all these points note $T_e > T_{fric}$ and the motor accelerates with the difference torque $T_e - T_{fric}$. The question is where the motor will finally settle down? The answer is it will settle finally at point P_o when the electromagnetic torque and the T_{fric} will be equal and the motor runs steadily at a speed very close to synchronous speed. From point P_o if we drop a perpendicular on the slip axis, the no load speed will be obtained (- for the sake of clarity this is not shown in the characteristic).

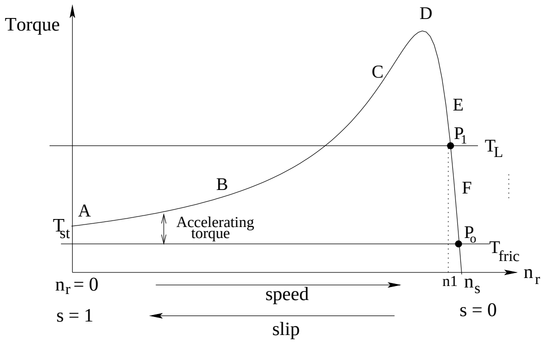


Figure 17: Operating point on T-s curve

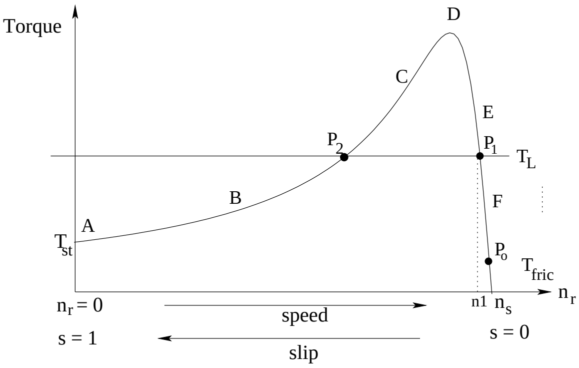


Figure 18: Stable and unstable zones

Now imagine, the opposing load torque is suddenly increased to a level T_L. Speed hence slip of the motor can not change instantaneously. Therefore $T_e < T_L$ and the motor starts decelerating and operating point starts moving from P_o to P_1 when once again motor rotates at constant speed n_1 and new operating point will be at P_1. The motor is now loaded and will run at slightly reduced speed compared to no load speed.

3.4.2 Unit-4 Problems

1. What is *synchronous watt*? What does it represent?
2. Sketch a typical torque-slip characteristic of a 3-phase induction motor and mark the axis clearly showing the range of slip values.
3. A 3-phase slip-ring induction motor has rotor resistance 0.2 Ω per phase and standstill rotor leakage reactance 2 Ω per phase. Stator impedance may be neglected. Calculate the value of slip for which (i) gross mechanical power developed will be maximum and (ii) gross torque developed will be maximum.

3.5 Unit-5 More on Torque-slip characteristic

Overview

Clear understanding of the nature of torque-slip characteristic is essential. The stable and unstable zone of the characteristic is identified. Expression for maximum torque and the slip at which maximum torque is obtained, are derived. This will help us to redraw the modified torque versus slip characteristics when say supply voltage is varied, rotor resistance is varied or voltage is varied keeping V/f ratio constant.

Objectives

After going through this unit, students will be able to:

1. identify the range of slip in which stable operation is ensured.
2. spell out the order of the full load slip.
3. conclude variation of speed from no load to full load condition is little.
4. sketch modified T-s characteristic when rotor resistance is increased and conclude that starting torque increases.
5. calculate the extra rotor resistance needed to have desired starting torque.
6. suggest how to control speed by (i) varying rotor resistance, (ii) by varying supply voltage and (iii) by varying supply frequency.

3.5.1 Suggested material to meet the objectives

Modified torque slip characteristic when rotor resistance is varied

For a slip ring machine we can connect external resistance in the rotor circuit which therefore changes the rotor resistance r_2'. The torque-slip characteristics of the motor for increasing rotor resistances are shown in figure 19. As we know maximum torque is independent of rotor resistance, so it remains same even if rotor resistance is varied. But the slip at which the maximum torque occurs becomes more with higher rotor resistance. Because of this, the characteristic shifts to the right with higher value of rotor resistance. Another interesting point to note that incorporation of external resistance in the rotor, increases the starting torque. In fact one can choose an external resistance which will make starting torque equal to the maximum torque the motor is capable of developing.

Modified torque slip characteristic when supply voltage is varied

Since T_{max} is directly proportional to the square of the supply voltage, T_{max} will become 1/4th if supply voltage is halved. Family of torque slip characteristics are shown in figure 20.

Modified torque speed characteristic when both supply voltage and frequency are varied keeping the ratio $\frac{V}{f}$ constant

. If the supply frequency is varied, synchronous speed hence rotor speed can be varied. For example for 4-pole, 50 Hz induction motor, synchronous speed is 1500 rpm. If this motor is supplied from a 25 Hz supply, the synchronous speed will become 750 rpm. However, to keep the flux level constant, decrease in supply frequency must be accompanied by a proportionate decrease in supply

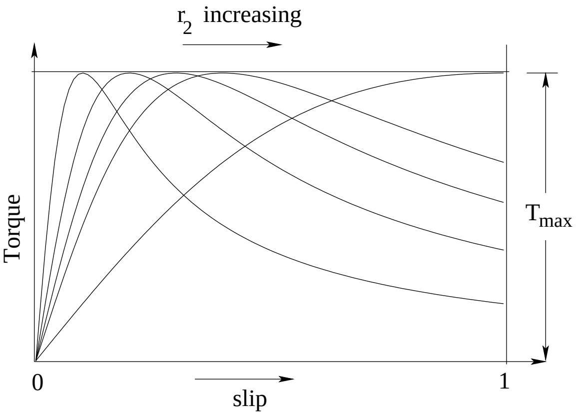


Figure 19: Effect of rotor resistance variation on torque-slip characteristic

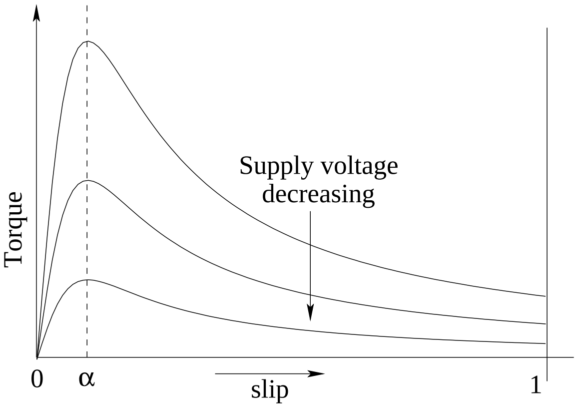


Figure 20: Effect of supply voltage variation on torque-slip characteristic

voltage i.e., V/f ratio should be maintained constant. In this case, since synchronous speed itself is changing, it is customary to plot torque-speed characteristic as shown in figure 21. As frequency is decreased, synchronous speed too decreases and the characteristic shifts to left. Recall that $T_{max} = V_1^2/2\pi n_s x_2' \ \alpha \ (v_1/f)^2$, so maximum torque practically remains constant. However due to stator resistance, T_{max} decreases at lower value of frequency.

3.5.2 Unit-5 Problems

1. Maximum torque of a 3-phase induction motor is obtained at a slip value of α. What is the range of slip for which stable operation will take place?
   Solution: Stable operation is possible when slip is in the range $0 < s < \alpha$.

2. How is it possible to make the starting torque same as the maximum torque in a wound rotor induction motor?
   Solution: By connecting suitable external resistance in the rotor circuit, starting torque may be made equal to maximum torque.

3. One wants to run a 3-phase, 4-pole, 50 Hz, 400 V induction motor, at half the rated speed. What should be the applied voltage and frequency?
   Solution: Applied voltage and frequency should be 200 V, 25 Hz. V/f ratio should be kept constant in order to avoid saturation.

4. For a 3-phase, 50 Hz, induction motor maximum torque occurs at a speed of 1350 rpm and full load torque is half the maximum torque. Estimate the full load speed.

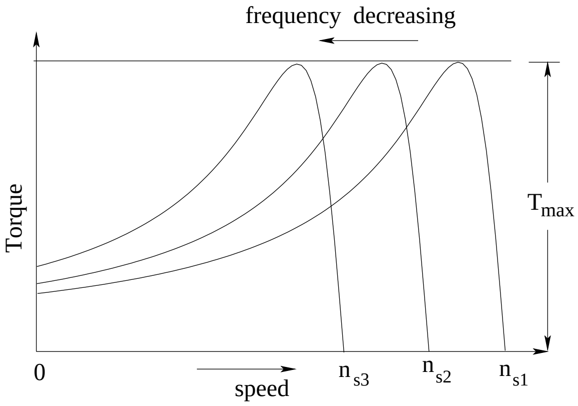


Figure 21: Effect of supply voltage frequency variation on torque-speed characteristic

Solution: For the given motor,

$$\begin{aligned} \text{Synchronous motor, } n_s &= 1500\,\text{rpm} \\ \text{Slip at which maximum torque occurs, } \alpha &= (1500 - 1350)/1500 = 0.01 \end{aligned}$$

In absence of any other data, a fair assumption that the torque-slip characteristic is linear in the stable zone, can be made.

$$\begin{aligned} T_{fl}/T_{max} &= s_{fl}/\alpha \\ s_{fl} &= \frac{1}{2} \times 0.01 = 0.05 \end{aligned}$$