BASIC ELECTRICAL ENGINEERING

Notes

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Basic Electrical Engineering

Module I (10 hours)

1.1 Introduction:

Electrical Engineering have contributed significantly to technical achievements during twentieth century. These contributions have resulted in the widespread use of electric energy and the advantages it affords.

Electric energy has a number of desirable attributes not possessed by alternative energy forms. Among these attributes are the ease with which electric energy is converted to and from other forms of energy, the ability to distribute it over large areas, and the speed with which it is transported.

Economics result when large quantities of energy are generated at one location and distributed to sites where the energy is needed. Ease of transportation to geographically separate regions is a significant factor in realizing these economics. Information, in the form of electric signals, can be propagated at great speed. As a result the effectiveness of long-range communication systems, data-processing systems, and control systems is enhanced. In many cases it is the speed at which the energy is transferred that makes the system feasible.

As an end in itself, electric energy is not generally useful. In most applications, other forms of energy are converted to electric energy at the input and reconverted at the output. Between input and output, the electric energy is processed and transmitted to the desired location in a manner appropriate for use. Motors are converters which provide mechanical energy to operate the machinery in a factory.

In turn, the electric energy to drive the motor is derived by converting mechanical energy in a steam-driven turbo generator at the power plant. Similarly, the information in the sound and picture in a television system is converted to electric signals at the studio by means of microphones and vidicons. Reconversion to optical and acoustical information in the receiver is accomplished by cathode-ray tubes and loud speakers. The availability of devices that reading perform the interconversion of energy is crucial to the use of electric energy. These conversion devices, coupled with the generally small size of electric components, form the basis for the use of electric energy in a variety of control, communication, and instrumentation systems.

The study of electrical engineering becomes one of investigating the characteristics and uses of devices and systems of energy conversion, processing, and transfer. Such devices and systems impact on all professional branches of engineering and science. Most instrumentation and control systems are, in part, electrical or electronic in nature. The incorporation of the digital computer in such systems increases the utilization of electrical instrumentation. A variety of engineers deal with motors and power distribution in building systems, process control, and design of manufacturing facilities. Medical scientists use electric devices in diagnostics, in prosthetics, and for environmental control. The aforementioned applications have, in turn, a vital impact on the economic, structural, and behavioral aspects of organizations and are, therefore, of vital concern to managers and executives.

For convenience, a basic study of electrical engineering is traditionally divided into treatments of circuit theory, of electronic devices, of energy conversion and electromechanical conversion devices, and of control devices and systems. This chapter will lag the groundwork for such a study by developing the elementary concepts and defining the basic terms. The typical reader already will have encountered much of this material. Thus, for many, it will simply be a coordinated review reflecting the general viewpoint of the remainder of the book.

As a core branch, electrical engineering has a wide range of core subjects, yet it is observed that this particular subject has applications in all branches of engineering.

In this subject, the use of SI system of units is extensively adopted for sake of convenience of the readers.

In this subject units of physical quantities are most important.

The following main topics will be covered by this subject. They are namely

(i) DC networks
(ii) Magnetic Circuits
(iii) Single phase AC circuits

(iv) Three phase AC circuits
(v) Transformers
(vi) DC machines
(vii) Induction motors
(viii) Measuring Instruments
(ix) Power systems

The Basic Electrical Engineering subject is to introduce the non-electrical engineering student to those aspects of electrical engineering that are likely to be most relevant to his or her professional career.

The possible interconnection of electrical engineering towards various disciplines are given below.

(i)Circuit analysis

(ii) Electromagnetic

(iii) Solid-state electronics

(iv) Electric machines

(v) Electric power systems

(vi) Digital logic circuits

(vii) Computer systems

(ix) Communication systems
(x) Electro-optics
(xi) Instrumentation systems
(xii) Control systems.

Basically Electrical Engineering deals with the following foundations or applications.

They are (1) Physical foundations

(2) Mathematical foundations

(3) Engineering applications.

(1) Physical Foundations:-

(i) Electromagnetic

(ii) Solid-state physics

(iii) Optics

(i) Electromagnetic(application)
(a) Power systems
(b) Electric machinery
(c) Communication systems
(d) Instrumentation system.

(ii) Solid –state physics (applications)
(a) Power systems
(b) Analog electronics
(c) Digital electronics
(d) Instrumentation systems.

(iii) Optics
(a)communication systems
(b) Instrumentation systems

(2) Mathematical foundations:

(i)Network theory

(ii) Logic theory

(iii) System theory

(i) Network Theory

(a) Power systems
(b) Electric machinery
(c) Analog electronics

(ii) Logic theory
(a) Digital electronics

(b) Computer systems

(iii) System theory

(a) Control systems

(b) Communication systems

(c) Instrumentation system.

(3) Engineering Applications:-

(a) Power systems

(b) Electric machinery

(c) Analog electronics

(d) Digital electronics

(e) Computer systems

(f) Control systems

(g) Communication systems

(h) Instrumentation systems.

(1.1) Electricity

It is a form of energy that depends on the existence of electric charge in static or dynamic form. As per the existence of electric charge, matter may be classified as positively charged, negatively charged and neutral.

Matter/Substance

↓

Molecules

↓

Atoms

↓

Particles(electron, proton & neutron

(1.2) Electric charge

It is a property of every matter that is responsible for the electrical phenomenon existing in the matter in a positive or negative form.

The unit of charge is coulombs(C).

(1.3) Fundamental charge

Electron , proton & neutron.

Electron charge (q_e)

q_e=e = -1.602x10^{-19} C

Proton charge (q_p)

q_p=-e = +1.602x10^{-19} C

Neutron (n)

Neutron is considered as charge less particle.

ELECTRICAL ENGINEERING

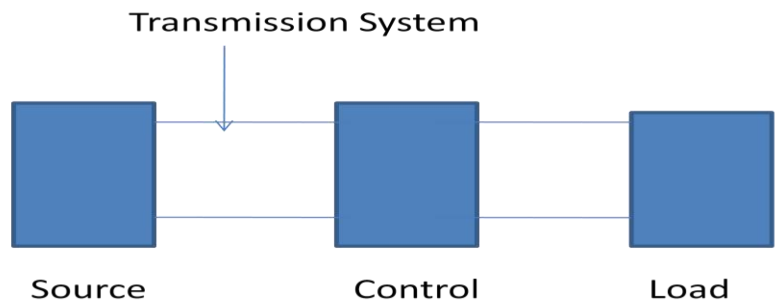
A basic electrical systems consists of four main parts,

1. **The Source**: The main part of a electrical system is source which provides energy for the electrical systems. A source may usually be a battery or a generator.

2. **The Load**: The function of load is to absorb the electrical energy supplied by source. e.g.; all domestic equipments like heaters, lamps etc.

3. **The Transmission Systems:** This conducts the energy from the source to the load. A transmission system consists of insulated wire.

4. **The Control Apparatus:** Its function is to control the electrical circuit equipments.



CHARGE:

Electric charge is the physical property of matter that causes it to experience a force. It is the excess or deficiency of electrons in the valance cell or outermost orbit of an atom. There are two types of electric charges: positive and negative. If an atom has an excess of electrons in the valance cell, then it is called negatively charged (-vely charged) and an atom with deficiency of electron is called positively charged (+vely charged).

Positively charged substances are repelled from other positively charged substances, but attracted to negatively charged substances; negatively charged substances are repelled from negative and attracted to positive. The SI derived unit of electric charge is the coulomb (C), although in electrical engineering it is also common to use the ampere-hour (Ah), and in chemistry it is common to use the elementary charge (*e*) as a unit. The symbol *Q* is often used to denote charge.

An electron is an elementary particle charged with a small and constant quantity of negative electricity. A proton is similarly defined but charged with positive electricity whereas neutron is uncharged and therefore neutral. In atom, the number of electrons normally equals the number of protons; it is the number of protons that determines to which element type the atom belongs. An atom can have one or more electrons added to it or taken away. This does not change its elemental classification but it disturbs its electrical balance. A charged atom is called ion. A body containing a number of ionized atoms is said to be electrically charged. Fig. 1 shows the behavior of likely charged and unlikely charged atoms.

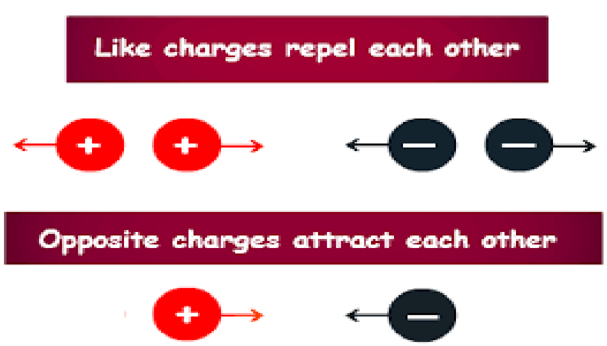


Fig. 1: The behavior of likely charged and unlikely charged atoms.

The charges of electron and proton are.

$$q_e = -1.602 \times 10^{-19}\, C$$

$$q_p = +1.602 \times 10^{-19}\, C$$

ELECTRIC CURRENT:

It is defined as the time rate of change of charge passing through a predetermined area with respect to time. It is caused by drift of free electrons through a conductor to a particular direction. The measuring unit of electric change is Coulomb and the unit of time is second, the measuring **unit of electric current** is Coulombs per second and this logical **unit of current** has a specific name Ampere after the famous French scientist André-Marie Ampere. Here the area is the cross-sectional area of the metal wire. If Δqunits of charge flowing through the cross sectional area A in Δt units of time, then the resulting current i is given by,

$$i = \frac{\Delta q}{\Delta t} \quad \text{C/S}$$

$$1\ Ampere\ (A) = \frac{1\ coulomb}{second} = \frac{Q}{t}$$

Let's have an example for better understanding. Suppose total 100 coulomb of charge is transferred through a conductor in 50 sec. What is the electric current?

Solution: As we know that, electric current is nothing but the rate at which charge is transferred per unit time. So, it would be the ratio of total charge transferred to the required time.

$$\text{electric current i} = \frac{100 \text{ coulomb}}{50 \text{ sec}} = 2 \text{ Ampere}$$

SI unit of measuring current= Ampere (A)

Symbol: I

The ampere is defined as that current which, if maintained in two straight parallel conductors of infinite length, of negligible circular cross-section, and placed 1 meter apart in a vacuum, would produce between these conductor a force of 2×10^{-7} Newton per meter of length.

Current Flow in a Circuit:

For most practical applications, it is necessary that current flow continues for as long as it is required; this will not unless the following conditions are fulfilled:

1. There must be a complete path around which the electrons may move. If the electrons cannot return to the point of starting, then eventually they will all congregate together and the flow will cease.
2. There must be a driving influence to cause the continuous flow. This influence is provided by source which causes the current to leave at a high potential and to move round the circuit until it returns to the source at a low potential.

The driving influence is termed as the electromotive force, hereafter called the e.m.f

Electro-motive Force (emf):

The e.m.f represents the driving influence that causes a current to flow. The e.m.f is not a force, but represents the energy expanded during the passing of a unit charge through the source.

The energy introduced into a circuit is transferred to the load unit by the transmission system, and the energy transferred due to the passage of unit charge between two points in a circuit is termed as the potential difference (p.d.). It will be observed that both emf and p.d. are similar quantity. However, an emf is always active in that it tends to produce an electric current in a circuit whereas a p.d. may be either passive or active. A p.d. is passive whenever it has no tendency to create a current in a circuit. Fig.2 shows the emf, p.d. and current flow in a circuit.

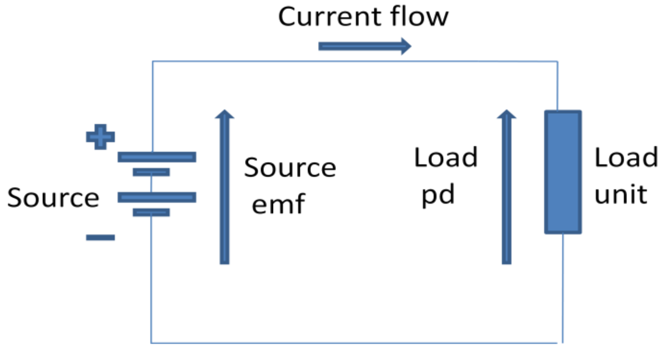


Fig. 2 The emf, p.d. and current flow in a circuit.

In this above figure, the arrowhead points towards the point of high potential. It is seen that the current flow leaves the source at the positive terminal and therefore moves in the same direction as indicated by the sources emf arrow. The current flow enters the load at the positive terminal, and therefore in the opposite direction to that indicated by the load p.d. arrow. Here the source indicated consists of a battery which delivers direct current, i.e. current which flows in one direction. An arrowhead is drawn on the transmission system to indicate the corresponding direction of conventional current flow.

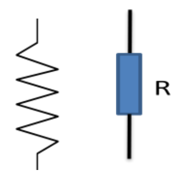
Electromotive force: Symbol- E, Unit- volt (V)

CIRCUIT ELEMENTS:

Mainly, there are three circuit elements present in the circuit such as, Resistance (R), Inductance (L) and Capacitance (C).

a) **RESISTANCE:**

It is defined as the property of a substance due to which it opposes or restricts the flow of electricity through it. The inverse of resistance is conductance, the ease with which electric current passes. Metals, acids and salt solutions are good conductor of electricity.

Symbol: R Unit: Ohm (Ω)and looks like- 

A conductor is said to have a resistance of 1 ohm if it permits 1 ampere current to flow through it when 1 volt is impressed across its terminals. The resistance R offered by a conductor depends on the following factors:

(i) It varies directly as its length, l.

(ii) It varies inversely as the cross-section A of the conductor.

(iii) It depends on the nature of the material.

(iv) It also depends on the temperature of the conductor.

So, according to above factors, resistance can be defined as,

$$R \alpha \frac{l}{A}$$

Or, $R = \rho \frac{l}{A}$

Where, ρ= a proportionality constant depending on the nature of the material of the conductor and is known as its **specific resistance or resistivity**.

Unit of resistivity ρ = $\frac{A\,R}{l}$

$$= \frac{A\,meter^2 \times R\,ohm}{l\,meter} = \frac{A\,R}{l}\,ohm - meter$$

Hence unit of resistivity is ohm-meter (Ω-m).

Conductance (G) is reciprocal of resistance.

$$G=\frac{1}{R}\text{Siemens}$$

Power P= $I\,V = I^2\,R = \frac{V^2}{R}$Watt

Energy dissipated is given by,

$$W= Pt= I^2\,R\,t = IVt \text{ Joule}$$

Example No.1

A current of 5 A flows in a resistor of resistance 8 Ω. Determine the rate of heat dissipation and also the heat dissipated in 30s.

Solution: $P = I^2\,R = \,5^2 \times 8 = 200\,W$

$$W=Pt= 200\times 30 = 6000\,J$$

Example No.2

A coil consists of 2000 turns of copper wire having a cross-sectional area of 0.8 mm^2. The mean length per turn is 80 cm and resistivity of copper is 0.02 μΩ-m. Find the resistance of the coil and power absorbed by the coil when connected across 110 V D.C. supplies.

Solution: Length of the coil, l= 0.8 × 2000 = 1600 m

$$A= 0.8\,mm^2= 0.8 \times 10^{-6} mm^2$$

$$R= \rho\,\frac{l}{A} = 0.02 \times 10^{-6} \times \frac{1600}{0.8 \times 10^{-6}} = 40\Omega$$

$$\text{Power absorbed, } P= \frac{V^2}{R} = \,\frac{110^2}{40} = 302.5\,W$$

OHM'S LAW:

One of the most important steps in the analysis of the circuit was undertaken by George Ohm, who found that p.d. across the ends of many conductors is proportional to the current flowing between them in 1827.

"The ratio of potential difference (V) between any two points on a conductor to the current (I) flowing between them, is constant, provided the temperature of the conductor does not change."

$$V \,\alpha\, I$$

$$\text{Or, } V= R\,I$$

$$\text{Or, } \frac{V}{I} = \text{Constant, R}$$

Where, R is the resistance of the conductor between two point considered. Fig.3 shows the parameters of Ohm's law.

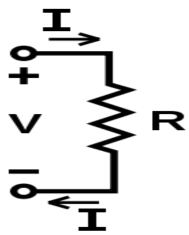


Fig.3 Ohm's Law

The three equivalent equations of Ohm's law are used interchangeably.

$I = \frac{V}{R}$ $or\ V = IR\ or\ R = \frac{V}{I}$

The interchangeability of the equation may be represented by a triangle in fig. 4, where 'V' is placed on the top section, 'I' is placed to the left section, and the 'R' is placed to the right. The line that divides the left and right sections indicate multiplication, and the divider between the top and bottom sections indicates division (hence the division bar).

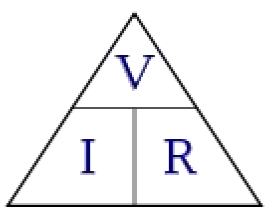


Fig. 4 Interchangeability Triangle

The V-I characteristic of resistance is shown in fig.5.

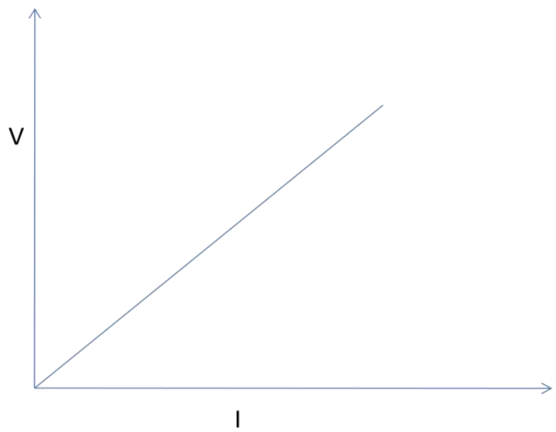


Fig.5 The V-I characteristic of resistance

SERIES CONNECTION OF RESISTANCES:

When some conductors having resistances R_1, R_2 and R_3 etc. are joined end-on-end as shown in fig.6, they are said to be connected in series. It can be proved that the equivalent resistance or total resistance between points A and D is equal to the sum of the three individual resistances. Being a series circuit, it should be remembered that,

(i) current is the same through all the three resistances,

(ii) but voltage drop across each is different due to its different resistance according to Ohm's law,

(iii) Sum of the three voltage drops is equal to the voltage applied across the three conductors.

(iv) There is a progressive fall in potential as we go from point A to point D.

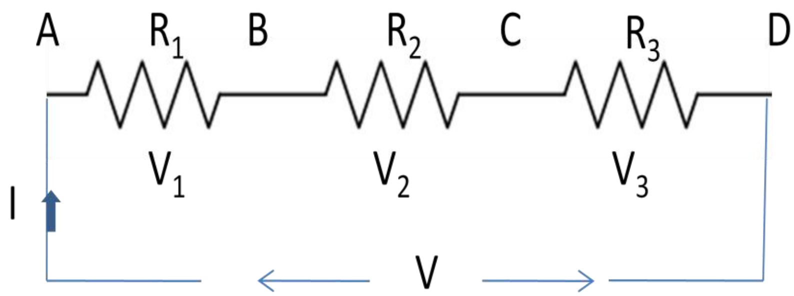


Fig 6 Series connection of resistances

So, $V = V_1 + V_2 + V_3 = IR_1 + IR_2 + IR_3$

Or, $R = R_1 + R_2 + R_3$

Main things to be remembered for series circuit are:

1. Same current flows through all parts of the circuit.
2. Different resistors have their individual voltages drops.
3. Voltage drops are additive.
4. Applied voltage equals the sum of different voltage drops.
5. Resistances are additive.
6. Powers are additive.

Total/ Equivalent Resistance > Greatest Resistance

PARALLEL CONNECTION OF RESISTANCES:

When three resistances are arranged together such that their starting points are connected together and ending points are connected together, then they are said to be connected in parallel as shown in Fig. 7. In this case,

(i) p.d. across all resistances is the same;

(ii) current in each resistor is different according to Ohm's law;

(iii) the total current is sum of the three separate currents.

Resistors are said to be connected together in "Parallel" when both of their terminals are respectively connected to each terminal of the other resistor or resistors. Unlike the previous series resistor circuit, in a parallel resistor network the circuit current can take more than one path as their are multiple nodes. Then parallel circuits are current dividers.

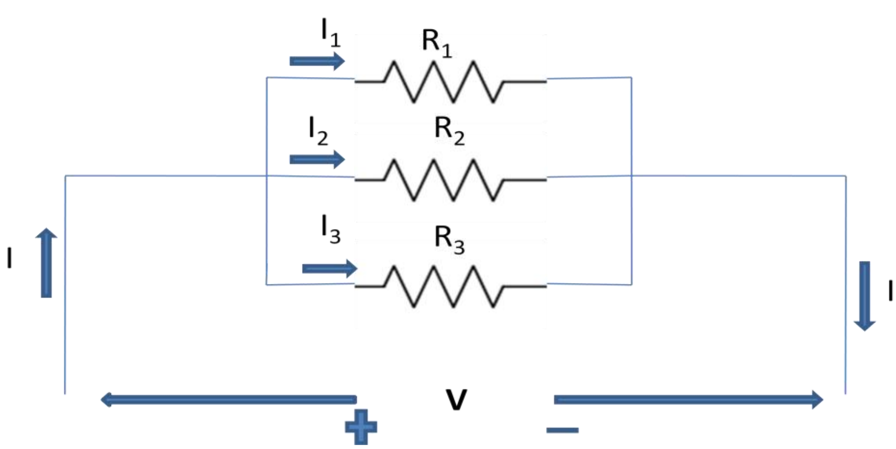


Fig 7 parallel connection of resistances

$I = I_1 + I_2 + I_3 = \frac{V}{R1} + \frac{V}{R2} + \frac{V}{R3}$

I= V/R where V is the applied voltage.

R= equivalent resistance of the parallel combination.

$$\frac{V}{R} = \frac{V}{R1} + \frac{V}{R2} + \frac{V}{R3} \quad or \quad \frac{1}{R} = \frac{1}{R1} + \frac{1}{R2} + \frac{1}{R3}$$

$VR_1 = VR_2 = VR_3 = V$

The main characteristics of parallel circuits are:

1. Same voltage acts across all parts of the circuits.
2. Different resistors have their individual current.
3. Branch currents are additive.
4. Powers are additive.

Total/Equivalent Resistance < Smallest Resistance of

Example No.3

What is the value of the unknown resistor R in Fig.8 if the voltage drop across the 500 Ω resistor is 2.5 volts? All resistances are in ohm.

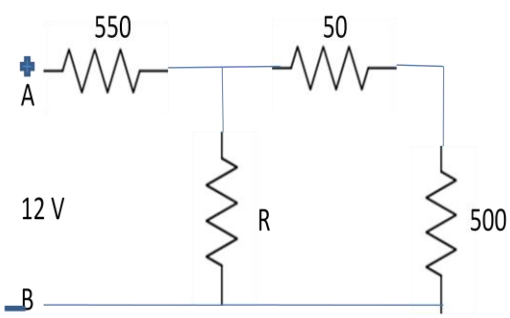
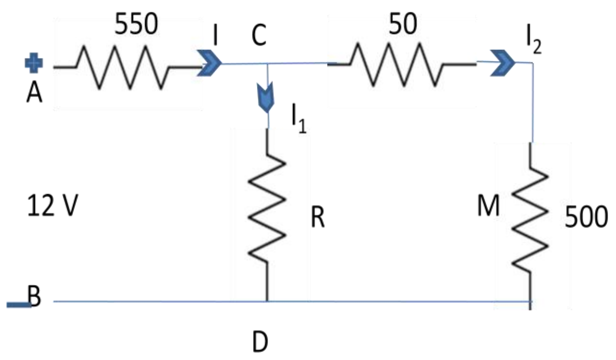


Fig.8

Solution:

Now the current through 500Ω is $\frac{2.5}{500}$ Amp

Voltage drop across 50 Ω resistor = $50 \times \frac{2.5}{500} = 0.25\,V$



Drop across CMD terminals or CD terminal = 2.5 + 0.25= 2.75 V

Drop across 550Ω resistance= 12- 2.75= 9.25 V

Current through 550Ω is I

Now I= 9.25/ 550= 0.0168 A

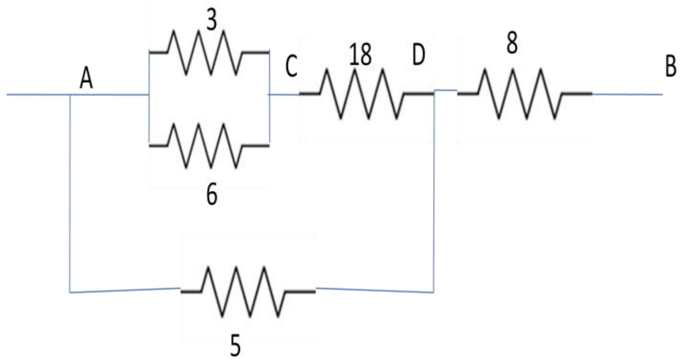
I_2 = 2.5/500= 0.005 A

I_1 = 0.0168- 0.005= 0.0118 A

R=2.75/0.0118=233Ω (Ans)

Example No.4

Calculate the effective resistance of the following combination of resistances and the voltage drop across each resistance when a p.d. of 60 V is applied between points A and B?



Solution: Resistance between A and C= 6 || 3= $\frac{6X3}{6+3} = 2\Omega$

Resistance of branch ACD= 18+ 2= 20Ω

Now, there are two parallel paths between points A and D of resistance 20Ω and 5Ω.

Equivalent resistance between A and D terminal= 20 || 5=(20X5)/(20+5)=4Ω

Net resistance between A and B terminal = 4+8 = 12Ω

Total circuit current= 60/12= 5 A

Current through 5 Ω resistance= $5\times \frac{20}{25} = 4\ A$

Current through branch ACD= $5 \times \frac{5}{25} = 1A$

Therefore, p.d. across 3Ω and 6Ω resistors=$1\times 2 = 2\ V$

P.d. across 18 Ω resistor= 1× 18= 18 V

P.d. across 5Ω resistor= 4 × 5= 20 V

P.d. across 8 Ω resistor= 5× 8= 40 V

VOLTAGE DIVIDER RULE:

Since in a series circuit, same current flows through each of the given resistor, voltage drop varies directly with its resistance. In Fig.8 is shown a 24 V battery connected across a series combination of three resistors.

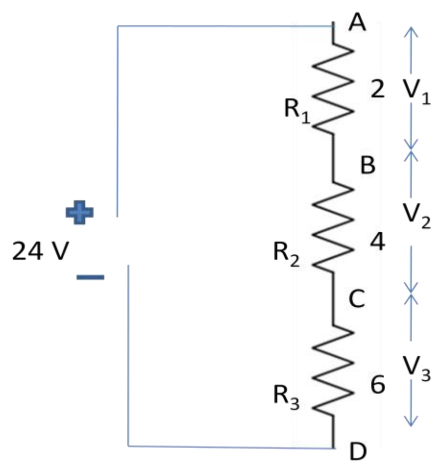


Fig. 8 Voltage Division Rule

Total resistance, R = R1 + R_2 + R_3= 12 Ω

According to voltage divider rule, voltage drops are:

$$V_1 = V\,.\frac{R1}{R} = 24 \times \frac{2}{12} = 4\ V$$

$$V_2 = V.\frac{R2}{R} = 24 \times \frac{4}{12} = 8\ V$$

$$V_3 = V.\frac{R3}{R} = 24 \times \frac{6}{12} = 12\,V$$

CURRENT DIVISION RULE:

Since in parallel circuit, different currents flow in all the resistances, but voltage drop is same as supply voltage. In Fig.9, a 12 V battery is connected across the parallel combination of 2Ω and 4Ω resistances. I_1 and I_2 are the currents flowing through 2 Ω and 4Ω resistances respectively.

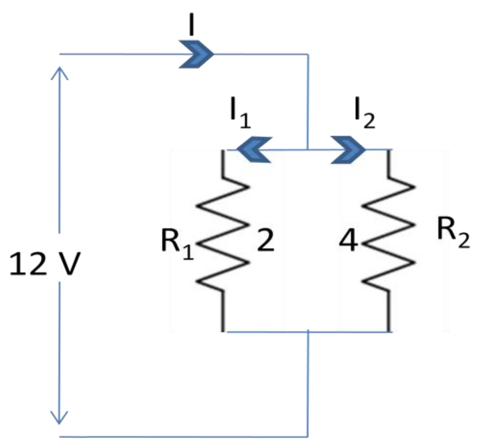


Fig.9 Current Division Rule

Total resistance, $R_1 + R_2 = 6\ \Omega$

Current through 2 Ω resistance, $I_1 = \frac{R2}{R1+R2}\ .I$

Current through 4 Ω resistance, $I_2 = \frac{R1}{R1+R2}\ .I$

So, current in any of the parallel branch is equal to the ratio of opposite branch resistance to the total resistance multiplied by the total current.

DUALITY BETWEEN SERIES AND PARALLEL CIRCUITS:

There is a certain peculiar relationship between series and parallel circuits. In series circuit, current is same whereas in parallel circuit, voltage is same. Also in a series circuit, individual voltages are added and in a parallel circuit, individual currents are added. Comparing series and parallel circuit, voltages takes the place of current and current takes the place of voltage. Such a pattern is known as "duality" and the two circuits are called to be duals of each other. Dual elements refer to same magnitude.

SERIES CIRCUIT	PARALLEL CIRCUIT
$I_1 = I_2 = I_3 = \ldots\ldots\ldots$	$V_1 = V_2 = V_3 = \ldots\ldots$

$V_{TOTAL} = V_1 + V_2 + V_3 \ldots\ldots$	$I_{TOTAL} = I_1 + I_2 + I_3 + \ldots\ldots$
$R_{TOTAL} = R_1 + R_2 + R_3 + \ldots\ldots$	$G_{TOTAL} = G_1 + G_2 + G_3 + \ldots\ldots$
$I = \frac{V1}{R1} = \frac{V2}{R2} = \frac{V3}{R3} = \ldots\ldots$	$V = \frac{I1}{G1} = \frac{I2}{G2} = \frac{I3}{G3} = \cdots$
Voltage Divider Rule $V_1 = V_{TOTAL.} \frac{R1}{RTOTAL}$	Current Division Rule, $I_1 = I_{TOTAL} \frac{G1}{GTOTAL}$

SHORT AND OPEN CIRCUITS:

When two points of circuit are connected together by a thick metallic wire, they are said to be short-circuited as shown in Fig.10. Since 'short' has practically zero resistances, it gives rise to two facts:

(a) No voltage can seen across it because $V = IR = I \times 0 = 0$ Volt.

(b) Current through it is very large, theoretically, infinity.

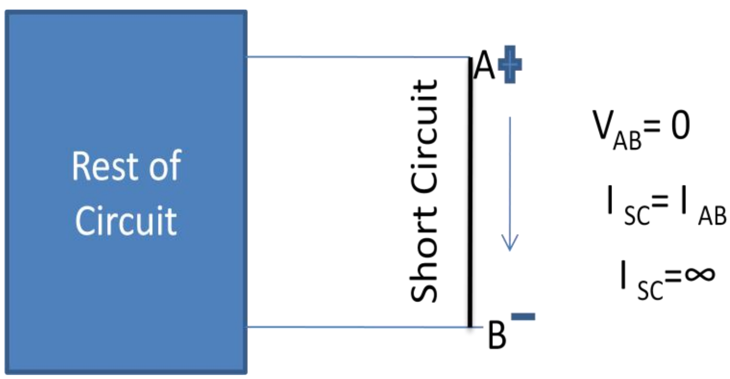


Fig.10 Short Circuit

Two points are said to be open-circuited when there is no direct connection between them. It is shown in Fig.11. An 'open' represents a break in the continuity of the circuit. Due to this, it has two facts;

(a) Resistance between the two points is infinite.

(b) There is no flow of current between these two points.

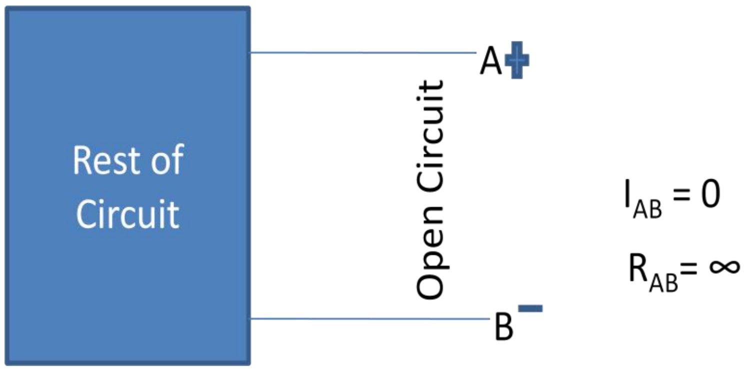
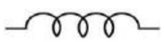


Fig.11 Open circuit

b) **INDUCTOR/ COIL/ REACTOR :**

It is the property of a material by virtue of which it opposes any change in direction or magnitude of electric current passing through the conductor.

Inductor-  Symbol- L Unit- Henry (H)

A wire of finite length when twisted into a coil becomes an inductor. When current of finite value flows through the inductor, an electromagnetic field is formed. So if any change in direction or flow of current occurs, then electromagnetic field also changes. This change in field induces a voltage across the coil. It is a passive two-terminal electrical component.

An inductor is characterized by its inductance, the ratio of the voltage to the rate of change of current, which has units of henries (H). Inductors have values that typically range from 1 µH (10^{-6}H) to 1 H. Many inductors have a magnetic core made of iron or ferrite inside the coil, which serves to increase the magnetic field and thus the inductance. Along with capacitors and resistors, inductors are one of the three passive linear circuit elements that make up electric circuits. Inductors are widely used in alternating current (AC) electronic equipment, particularly in radio equipment. They are used to block the flow of AC current while allowing DC to pass; inductors designed for this purpose are called chokes. They are also used in electronic filters to separate signals of different frequencies, and in combination with capacitors to make tuned circuits, used to tune radio and TV receivers.

Inductance (L) results from the magnetic field around a current-carrying conductor; the electric current through the conductor creates a magnetic flux. Mathematically speaking, inductance is determined by how much magnetic flux φ through the circuit is created by a given current.

$$L = \frac{d\varphi}{di}$$

Any change in the current through an inductor creates a changing flux, inducing a voltage across the inductor. By Faraday's law of induction, the voltage induced by any change in magnetic flux through the circuit is,

$$v = \frac{d\varphi}{dt}$$

$v = \frac{d}{dt}(Li) = L\,\frac{di}{dt}$ ('-'= change in the direction of current)

Inductance also a measure of the amount of emf generated for a given rate of change of current.

P= vi= L i $\frac{di}{dt}$

Energy stored, W= $\int_0^t P.dt = \int_0^t Li.\frac{di}{dt}\,dt = \frac{1}{2}\,L\,i^2$Joules.

So, inductor can store finite amount of energy. A pure inductor doesn't dissipate energy, but store it.

Notes:

- Inductor behaves as 'short circuit' in steady state when direct steady current flow through it.
- In inductor, current can't change abruptly.

(c)**CAPACITOR:**

It is otherwise called as 'condenser' is a passive two-terminal device which store electric charge within it. A capacitor stores electric energy in the form of electric field being established by the two polarities of charges, on the two electrodes of a capacitor. Fig.12 shows different types of capacitors.

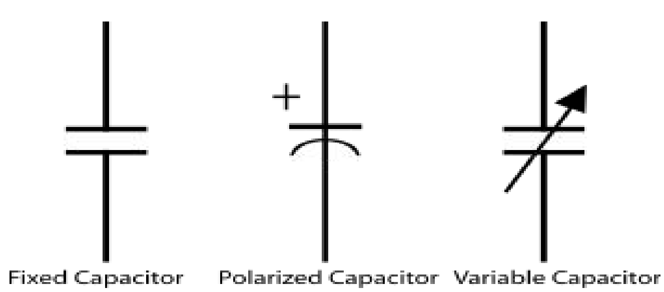


Fig.12 Capacitors

Symbol- C Unit- Farad (F)

An ideal capacitor is wholly characterized by a constant capacitance *C*, defined as the ratio of charge ±*Q* on each conductor to the voltage *V* between them.

$$C = \frac{Q}{V}$$

Because the conductors (or plates) are close together, the opposite charges on the conductors attract one another due to their electric fields, allowing the capacitor to store more charge for a given voltage than if the conductors were separated, giving the capacitor a large capacitance.

Differentiating above equation,

$$Q = CV$$

Or, $\frac{dQ}{dt} = C.\frac{dV}{dt}$

Or, i= C $\frac{dV}{dt}$ or, dV= $\frac{1}{C}\ i\ dt$

Or, $\int_{v0}^{vt} dV = \frac{1}{C}\int_0^t i.dt$

Or, $v_t - v_0 = \frac{1}{C}\int_0^t i\ dt$

Or, $v_t = \frac{1}{C}\int_0^t i\ dt + v0$

P= vi= vC$\frac{dV}{dt}$ Watt

Energy stored, W= $\int_0^t P.dt = \int_0^t vC\ \frac{dV}{dt}\ dt = \frac{1}{2}\ C\ V^2$ Joules.

Note:

- Capacitor behaves as a short circuit with no initial charge, when dc voltage is applied.
- Capacitor behaves as an open circuit as soon as it attains full charge.
- Capacitor never dissipates energy, only store it.

KIRCHHOFF'S CIRCUIT LAWS:

While considering series and parallel connections of resistors, we can see certain conditions appearing to each form of connections. For instant, in a series circuit, the sum of the voltages across each of the components is equal to applied voltage; again sum of the current in the branches of parallel network is equal to the supply current. Kirchhoff's circuit laws are two equalities that deal with current and potential difference. They are first described by Gustav Kirchhoff in 1845. This generalized the work of George Ohm. Widely used in electrical engineering and formally called Kirchhoff's Rules or Kirchhoff's Laws.

Kirchhoff's Current Law (KCL):

This law is otherwise called as **Kirchhoff's First Law** or **Kirchhoff's Point Rule** or **Kirchhoff's Junction Rule** (Nodal Rule).

It states that;

"In any electrical network, the algebraic sum of the current meeting at a point (or junction) is zero". Different signs are allocated to currents held to flow towards the junction and to those away from it.

So, total current entering a junction= total current leaving that junction. It means that, there is no accumulation of charge at any junction of the network. KCL indicates the law of conservation of charge. Positive signs are given for the currents entering to junction and negative currents are given to those who are leaving from that junction. It can be understood from Fig.13.

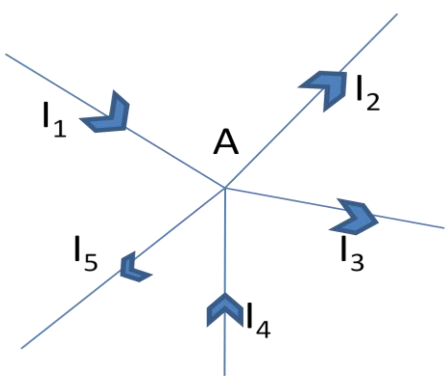


Fig.13 Kirchhoff's Current Laws

In this above figure, currents I_1 and I_4 are leading to point A, So positive sign will be assigned to them. But, currents I_2, I_3 and I_5 are leading away from point A. So negative sign will be given to them. By writing in equation form,

$$I_1 + (- I_2) + (- I_3) + (+ I_4) + (-I_5) = 0$$

$$\text{Or, } I_1+ I_4- I_2- I_3 - I_5 =0$$

$$\text{Or, } I_1+I_4 = I_2+ I_3 + I_5$$

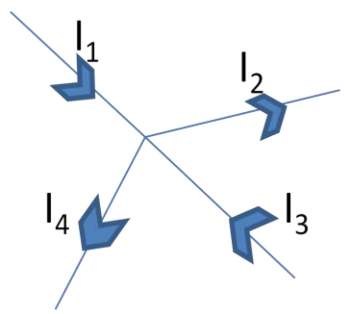
Incoming current= Outgoing current

We can express from above conclusion that,

$$\sum I = 0$$

Example No.5

For the network junction shown below, find out the current I_3, given that I_1= 3 A, I_2= 4 A and I_4= 2 A.



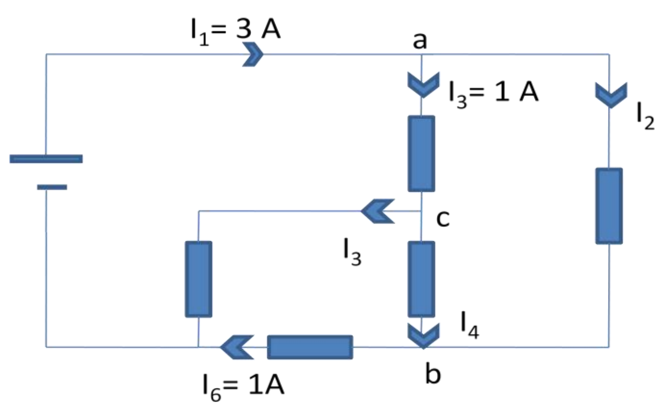
Solution: According to KCL, incoming currents= outgoing currents

$I_1 + I_3 = I_2 + I_4$

Or, $I_3 = -I_1 + I_2 + I_4 = -3+4+2 = 3$ A.

Example No.6

Write down the current relationship for junctions a, b and c of the networks shown in figure below and hence determines the current I_2, I_4 and I_5.



Solution: At junction a, I_1 is entering and I_2 , I_3 are leaving. So according to KCL:

$I_1 = I_2 + I_3$

Or, $I_2 = I_1 - I_3 = 3-1=2$ A

At junction b, I_2, I_4 are entering and I_6 is leaving. So,

$I_2 + I_4 = I_6$

Or, $I_4 = I_6 - I_2 = 1-2 = -1$ A

Here, the negative sign of I_4 indicates that the current is flowing in opposite direction i.e. from b to c. In the above diagram, we assumed the current I_4 direction as download, but the negative answer made our assumption wrong. That means the current is flowing in upward direction.

At junction c, I_3 is entering and I_4 and I_5 are leaving. So,

$$I_3 = I_4 + I_5$$

Or, $I_5 = I_3 - I_4 = 1+1 = 2$A

Kirchhoff's Voltage Law:

It is otherwise called as Kirchhoff's Second Law or Kirchhoff's mesh (loop) Rule. It states that,

"The algebraic sum of the products of currents and resistances in each of the conductor in any closed path in a network plus the algebraic sum of the emfs in that path is 0".

$$\sum IR + \sum emf = 0$$

This law is based on the conservation of energy whereby voltage is defined as the energy per unit charge. The total amount of energy gained per unit charge must be equal to the amount of energy lost per unit charge, as energy and charge are both conserved.

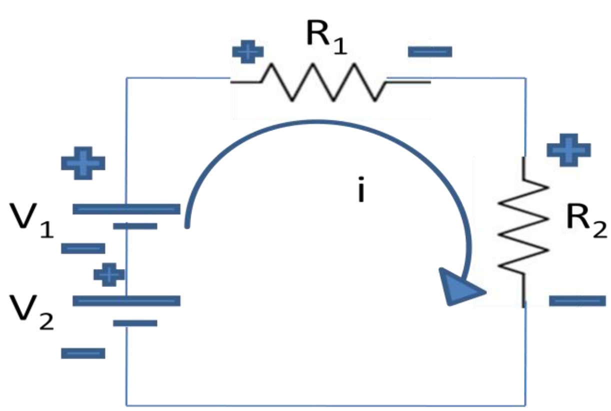


Fig. 14 Kirchhoff's Voltage Law

Applying KVL in whole loop in clockwise direction of current i as shown in Fig.14,

$$V_1 - i\,R_1 - i\,R_2 + V_2 = 0$$

Or, $V_1 + V_2 = i\ (R1+R_2)$

Or, Total voltage= Total potential difference

Electrical Power:

Electric power is defined as the rate of doing work. Electric power refers to the rating of an electric device and is defined as the rate at which the device can transform electrical energy into other forms of energy such as mechanical energy , heat energy , and light energy. It is worth noting that resistance is the major circuit parameter that consumes electrical energy and dissipates it in terms of heat and light. The power loss in resistors is generally referred as ohmic loss or copper loss.

It is a variable loss and mainly depends on the load current. For a purely resistive circuit, the expression for power may be written as $P=W/t=V^2/R=I^2R$.

Another way, we can express as

$P \propto V$

and $P \propto I$

Where P (Work done per unit time) $\propto$ VI (Voltage X Current)

The volt is the potential difference across a conductor when passing a current of 1 A and dissipating energy at the rate of 1 W. It follows that,

$\boxed{P= VI}$

Symbol- W , Unit- watt

In simple load condition, V= IR.

So , Power will be

$$\boxed{P= (IR)\ I= I^2R= \frac{V^2}{R}}$$

Example No.7

A 230 V lamp is rated to pass a current of 0.26 A. Calculate its power output. If a second similar lamp is connected in parallel to the lamp. Calculate the supply current required to give the same power output in each lamp.

Solution: Power P= VI= 230 × 0.26 = 60W

With the second lamp in parallel,

P= 60+60= 120 W

$I = \frac{P}{V} = \frac{120}{230} = 0.52\ A$

The relationship between real power (P), reactive power (Q) and apparent power (S) can be expressed by representing the quantities as vectors. Real power is represented as a horizontal vector and reactive power is represented as a vertical vector. The apparent power vector is the hypotenuse of a right triangle formed by connecting the real and reactive power vectors. This representation is often called as the *power triangle* as shown in Fig.15. Using the Pythagorean Theorem, the relationship among real, reactive and apparent power is:

$$(Apparent\ Power)^2 = (Real\ Power)^2 + (Reactive\ Power)^2$$

$$S^2 = P^2 + Q^2$$

Where,

Apparent power S= VI & Unit is Volt-Ampere or VA

Real power P=VIcos φ & Unit is Watts.

Reactive power Q=VIsin φ & Unit is Volt Ampere Reactive or VAR.

Real and reactive powers can also be calculated directly from the apparent power, when the current and voltage are both sinusoids with a known phase angle φ between them:

$$(Real\ Power) = (Apparent\ Power) Cos\ \varphi$$

$$(Reactive\ Power) = (Apparent\ Power) Sin \varphi$$

The ratio of real power to apparent power is called power factor and is a number always between 0 and 1. Where the currents and voltages have non-sinusoidal forms, power factor is generalized to include the effects of distortion.

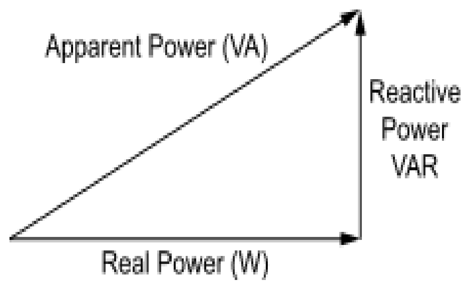


Fig.15 Power Triangle

SOURCE CONVERSION:

A given voltage source with series resistance can be converted into (or replaced by) an equivalent current source with a parallel resistance using Ohm’s Law. Remember that Ohm's law states that a voltage on a material is equal to the material's resistance times the amount of current through it (V=IR). Conversely, a current source with a parallel resistance can be converted into a voltage source with a series resistance. Specifically, source transformations are used to exploit the equivalence of a real current source and a real voltage source. We want to convert the voltage source of Fig.16 (a) into an equivalent current source.

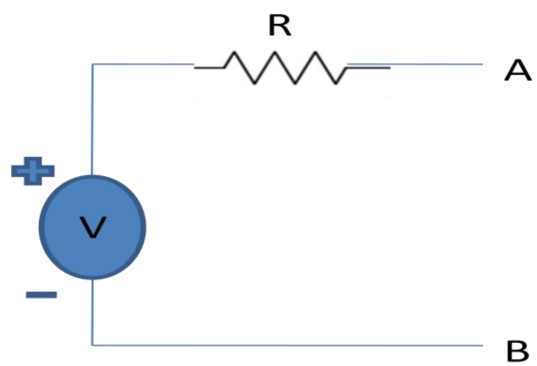


Fig.16(a) Voltage source with series resistance

First of all, we have to find out the current supplied by the source when a ‘short’ is put across in terminals A and B as shown in Fig.16 (b).

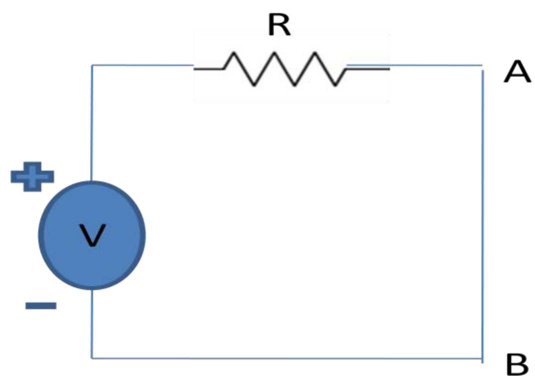


Fig.16(b) Voltage source with series resistance and short circuit

A current source supplying this current I and having the same resistance R connected in Parallel with it represents the equivalent source. It is shown in Fig. 16(c).

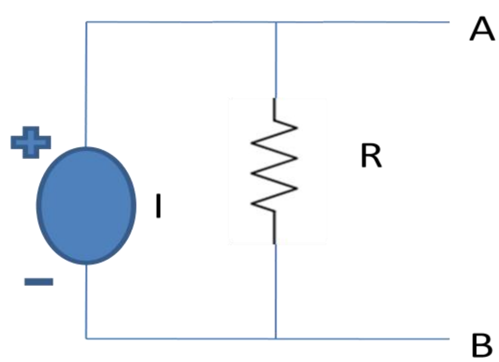


Fig.16(c) Current source with parallel resistance

Similarly, a current source of I and a parallel resistance R can be converted into a voltage source of voltage V= IR and a resistance R in series with it. This conversion can be done if and only if,

- Respective open-circuit voltage are same, and
- Respective short- circuit current are equal
- Resistance remains same in both cases.

So, source conversion can be summarized as in Fig.16(d) below.

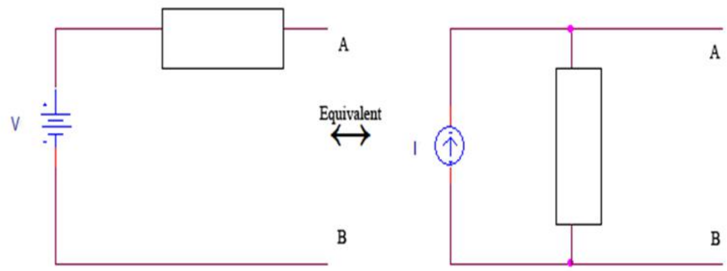
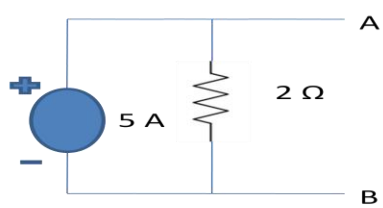


Fig.16(d)

Example No.8

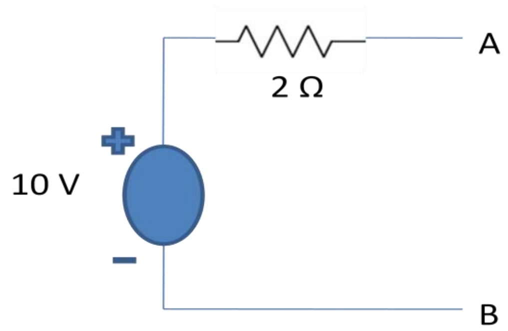
Find the equivalent voltage source for the current source in figure below.



Solution: The open- circuit voltage across terminals A and B in above figure is,

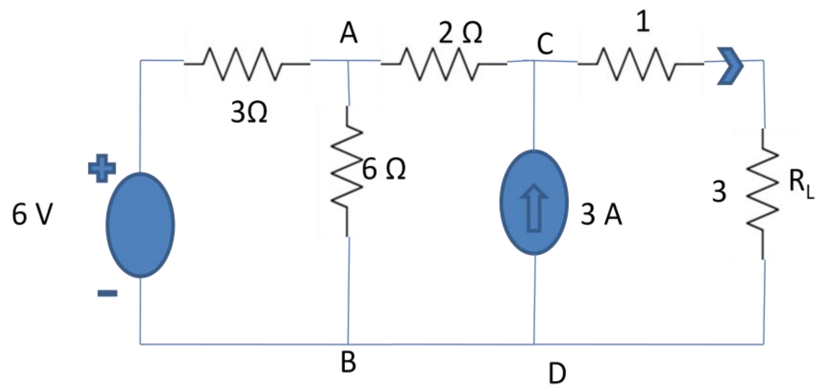
V_{OC} = drop across R= 5× 2= 10 V

Hence, voltage source has a voltage of 10 V and the same resistance of 2 Ω through connected in series.

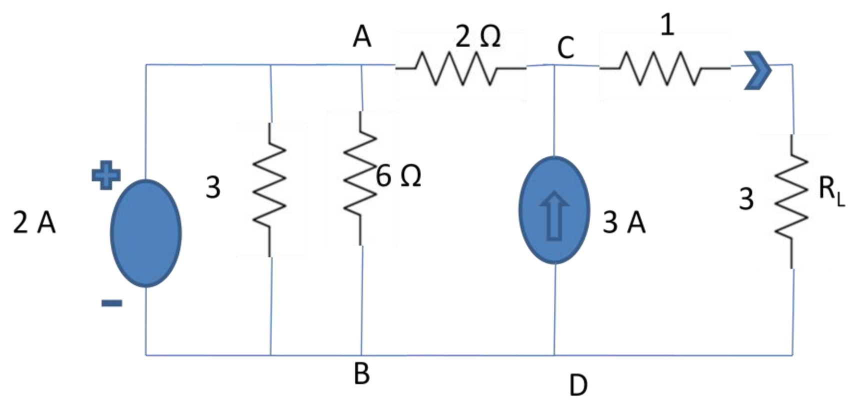


Example No.9

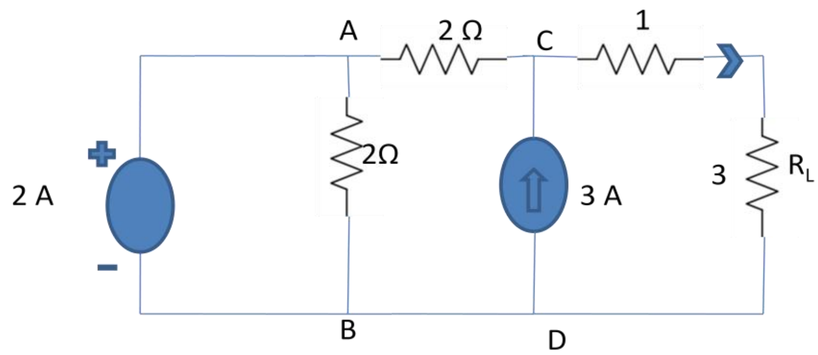
Using source conversion technique, find out load current through R_L in circuit given below?



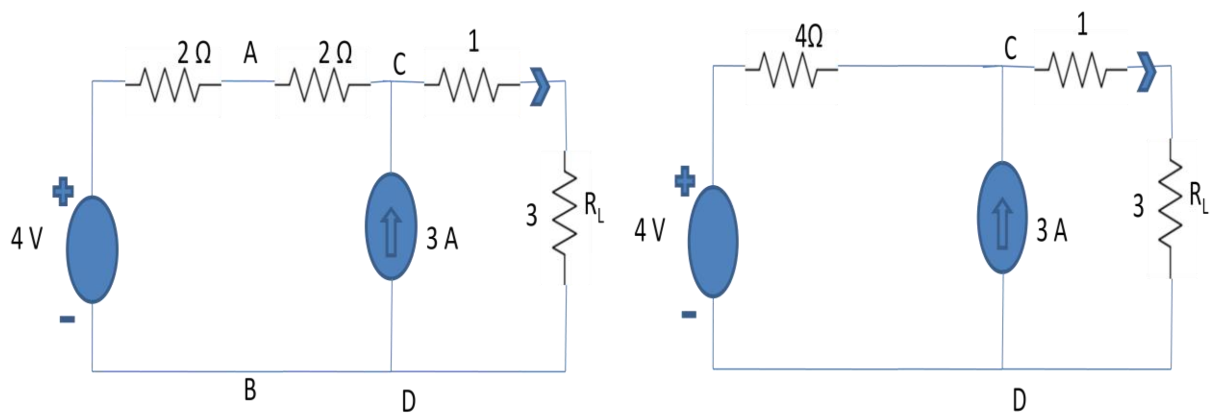
Solution: As shown in above figure, 6 V voltage source with a series resistance of 3 Ω has been converted into 2 A current source with 3 Ω resistance in parallel. By redrawing the circuit again,



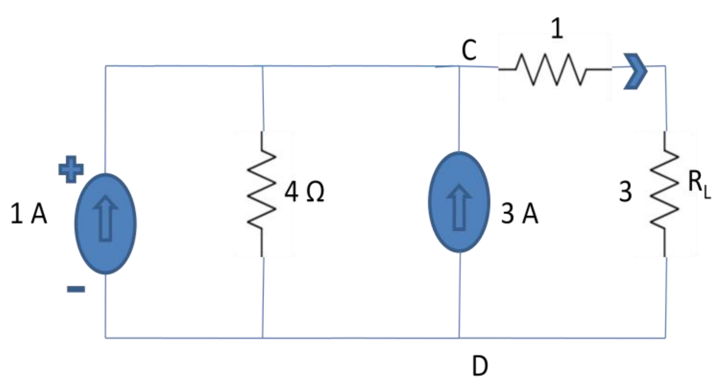
The two parallel resistances of 3Ω and 6Ω can be converted into equivalent resistance of 2 Ω.



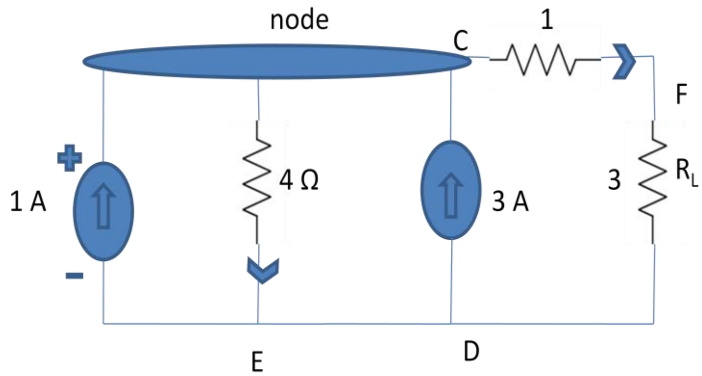
Now, the 2A current source with parallel resistance 2 Ω can be converted into equivalent 4V voltage source with 2Ω series resistance. Again, the series connection of two 2Ω resistance can be converted into one 4Ω resistance.



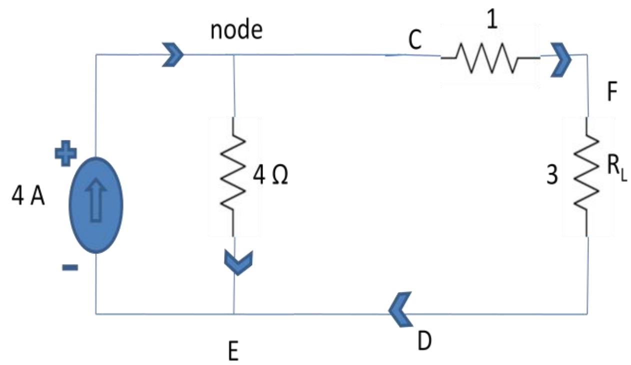
In the above figure, you can see that there is a series connection of 4V voltage source and 4Ω resistance. So, it can be converted into equivalent current source of 1A with parallel resistance of 1Ω.



Here, there are two current sources 1A and 3A entering into a node and leaving through CE and CFD path like below, then we can apply KCL here to find out current through R_L. So,



These two current sources can be added to become one source of value 4A.



So, 4A current is divided into two equal parts at node because each of the two parallel paths has a resistance of 4 Ω. I_{RL}= 2 A.

DELTA/STAR OR Π/ T TRANSFORMATION:

While solving networks by the application of Kirchhoff's Laws, one may sometimes experiences great difficulty due to a large number of simultaneous equations that have to be solved. However, such complicated networks can be solved by successively replacing delta meshes into equivalent star system and vice versa. Delta and star connections are shown in Fig.17.

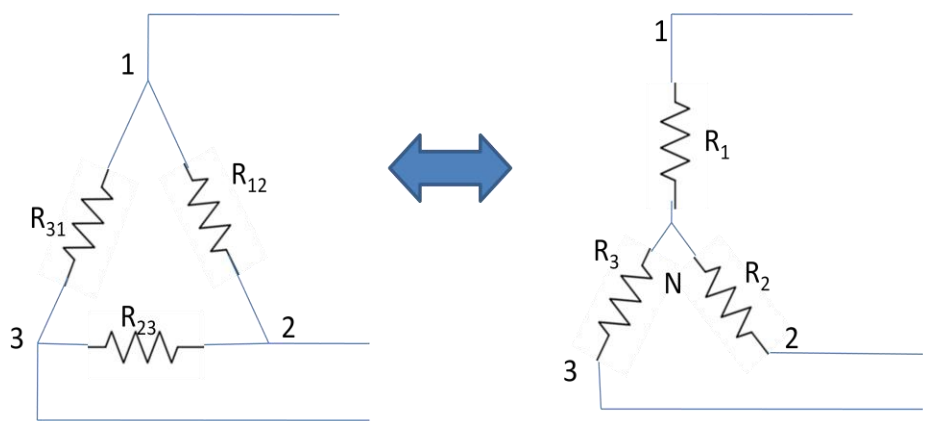


Fig.17 (a) Delta Form (b) Star form

Delta to Star Transformation:

Suppose three resistances R_{12}, R_{23} and R_{31} are connected in delta fashion between terminals 1, 2 and 3 as shown in above Fig.17. These given resistances can be replaced by the three resistances R_1, R_2 and R_3 connected in star like above Fig.17. These two arrangements will be electrically equivalent if the resistance as measured between any pair of terminals is the same in both the arrangements.

Let's take delta connection:

Between terminals 1 and 2, there are two parallel paths; one having a resistance of R_{12} and the other having a resistance of $(R_{23}+ R_{31})$.

Therefore, resistance between terminals 1 and 2 is = $\frac{R_{12}X(R_{23}+R_{31})}{R_{12}+R_{23}+R_{31}}$

Taking into account the star connection, the resistance between the same terminals 1 and 2 is $(R_1 + R_2)$.

As terminal resistance have to be the same,

$$R_1+R_2= \frac{R_{12}X(R_{23}+R_{31})}{R_{12}+R_{23}+R_{31}} \quad \text{------------------} \quad \text{(i)}$$

Similarly, for terminal 2 and 3 and terminal 3 and 1, we get

$$R_2+R_3=\frac{R_{23}X(R_{31}+R_{12})}{R_{12}+R_{23}+R_{31}} \quad \text{---------------} \quad \text{(ii)}$$

$$\text{And,} \quad R_3+R_1 = \frac{R_{31}X(R_{12}+R_{23})}{R_{12}+R_{23}+R_{31}} \quad \text{------------------} \quad \text{(iii)}$$

So, eq(i) – eq(ii) + eq(iii)= $R_1 = \frac{R_{12}R_{31}}{R_{12}+R_{23}+R_{31}}$

Like this, eq(ii)- eq(iii) +eq(i)= $R_2 = \frac{R_{23}R_{12}}{R_{12}+R_{23}+R_{31}}$ and

Eq(iii)- eq(i)+ eq(ii)= $R_3 = \frac{R_{31}R_{23}}{R_{12}+R_{23}+R_{31}}$

$R_1 = \frac{R_{12}R_{31}}{R_{12}+R_{23}+R_{31}}$, $R_2 = \frac{R_{23}R_{12}}{R_{12}+R_{23}+R_{31}}$ & $R_3 = \frac{R_{31}R_{23}}{R_{12}+R_{23}+R_{31}}$

Resistance of each arm of the star is given by the product of the resistances of the two delta sides that meet at its end divided by the sum of the three delta resistances.

Star to Delta Transformation:

$R_{12} = \frac{R_1R_2+R_2R_3+R_3R_1}{R3} = R_1 + R_2 + \frac{R_1R_2}{R_3}$

$R_{23} = R_2 + R_3 + \frac{R_2R_3}{R_1}$

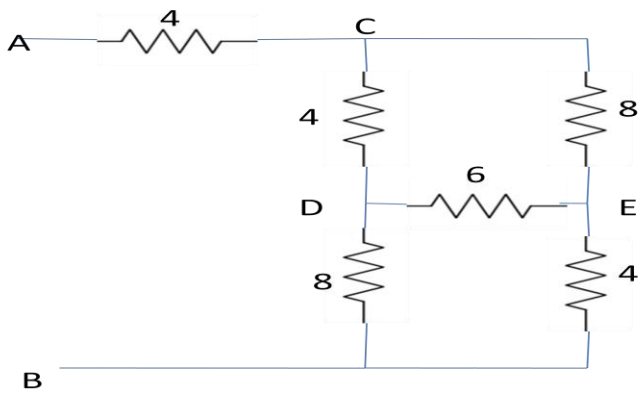
$R_{13} = R_1 + R_3 + \frac{R_1R_3}{R_2}$

The equivalent delta resistance between any two terminals is given by the sum of star resistances between those terminals plus the product of these two star resistances divide by the third star resistance.

$\boxed{R_{STAR} = R_{DELTA}}$

Example No.10

Find the input resistance of the circuit between the points A and B of below figure.



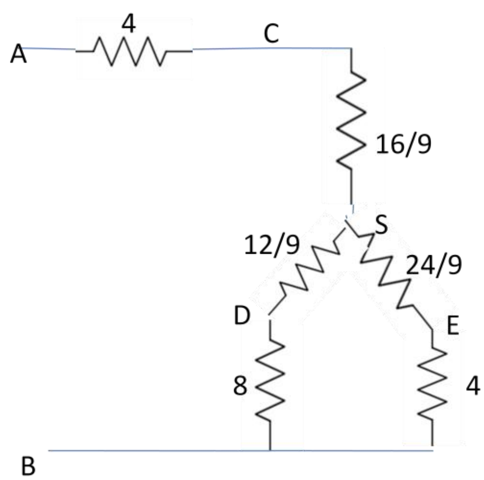
Solution: For finding out R_{AB}, we have to convert the delta CDE into its equivalent star. So,

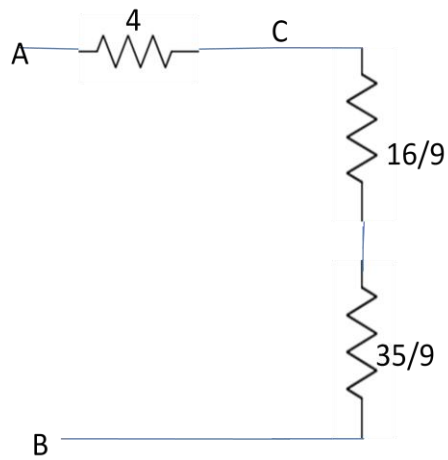
$$R_{CS}= \frac{8\times4}{18} = \frac{16}{9}\ \Omega$$

$$R_{ES}= \frac{8\times6}{18} = \frac{24}{9}\ \Omega$$

$$R_{DS}= \frac{6\times4}{18} = \frac{12}{9}\ \Omega$$

By redrawing the above circuit,



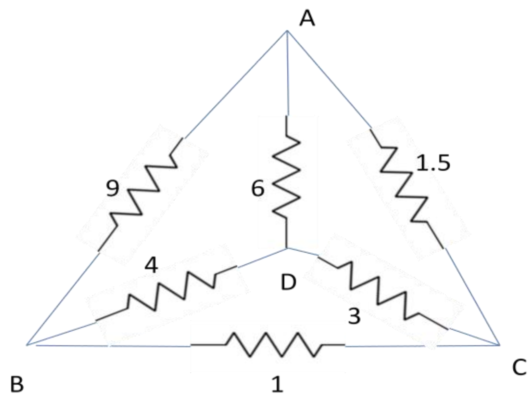


The two parallel path between S and B can be reduced to a single resistance of 35/9 i.e., (12/9 +8) || (24/9+4).

R_{AB}= 4+ 16/9+ 35/9= 87/9 Ω.

Example No.11

A network of resistances is formed as shown in the figure below.



Solution: There are two combinations of resistances in above figure i.e. inner star and outer delta. So, the inner star can be converted into equivalent delta and combined in parallel with the outer delta ABC.

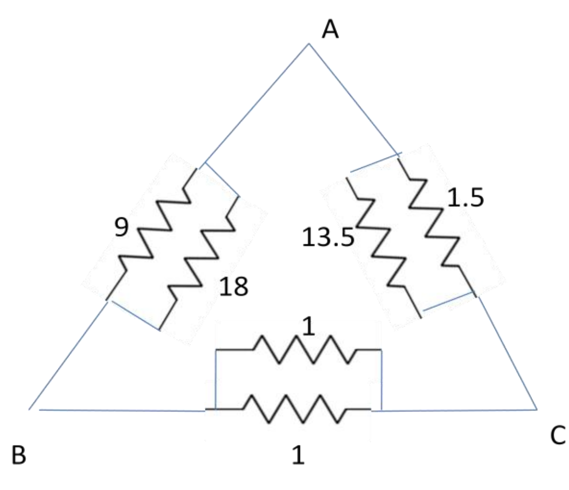
R_{AB}= 6+ 4+ $\frac{6\times4}{3}$ =18 Ω

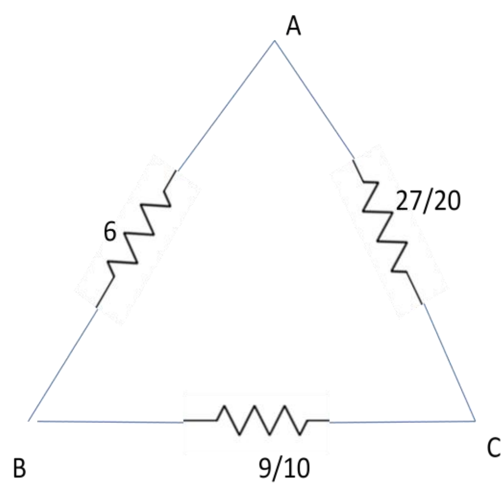
R_{AC}= 6+3+ $\frac{6\times3}{4}$ =13.5 Ω

R_{BC}= 4+3+ $\frac{4\times3}{6}$ =9Ω

So, the new figure can be as like this.

9Ω || 18Ω= 6Ω, 1.5 Ω || 13.5 Ω= 27/20 Ω and 1Ω || 1Ω= 9/10 Ω





R_{AB} is the resistance between terminals A and B means 6 || (27/20+ 9/10), R_{BC} is the resistance between terminals B and C i.e. 9/10 || (6+ 27/20) and R_{CA} is the resistance between terminals C and A i.e., 27/20 || (6+ 9/10).

$$R_{AB} = \frac{6\times 2.25}{6+2.25} = \frac{18}{11}\Omega$$

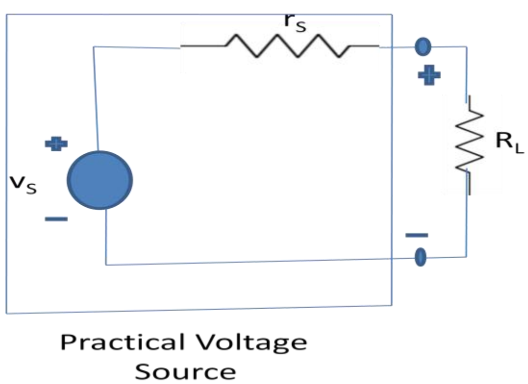
$$R_{BC} = \frac{\frac{9}{10}\times(6+\frac{27}{20})}{\frac{9}{10}+6+\frac{27}{20}} = \frac{441}{550}\,\Omega$$

$$R_{CA} = \frac{\frac{27}{20}\times(6+\frac{9}{10})}{\frac{9}{10}+6+\frac{27}{20}} = \frac{621}{550}\,\Omega$$

PRACTICAL VOLTAGE AND CURRENT SOURCES:

Practical Voltage Sources:

It is composed of ideal voltage source υ_s in series with resistance r_s in Fig.18 below.



$$i_s = \frac{vs}{rs+RL}$$

Fig.18 Practical Voltage Source

$$v_L = i_s.\,R_L = \frac{vs.RL}{rs+RL}$$

Practical Current Sources:

It consists of an ideal current source i_s with parallel resistance r_s as in Fig.19

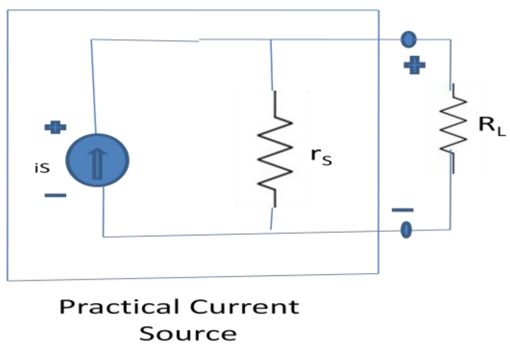


Fig.19 Practical current Source

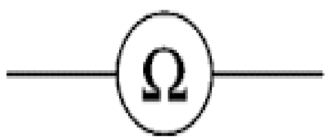
$v_{smax} = i_s r_s$

MEASURING DEVICES:

There are four measuring devices that are being used for the measurement of different parameters in the circuits. These devises are such as ohmmeter, ammeter, voltmeter and wattmeter or power-factor meter.

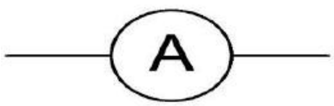
(a) Ohm-meter:

Ohmmeter is a measuring device when connected across a circuit element, can measure the electrical resistance of the element, the opposition of electric current. The resistance of an element can be measured only when the element is disconnected from any other circuit.

Symbolized as: 

(b) Ammeter:

Ammeter is a device that when connected in series with a circuit element, can measure the current flowing through the element. It must be placed in series with the element whose I is to be measured. The ideal ammeter has zero internal resistance.

Symbolized as: 

(c) Voltmeter:

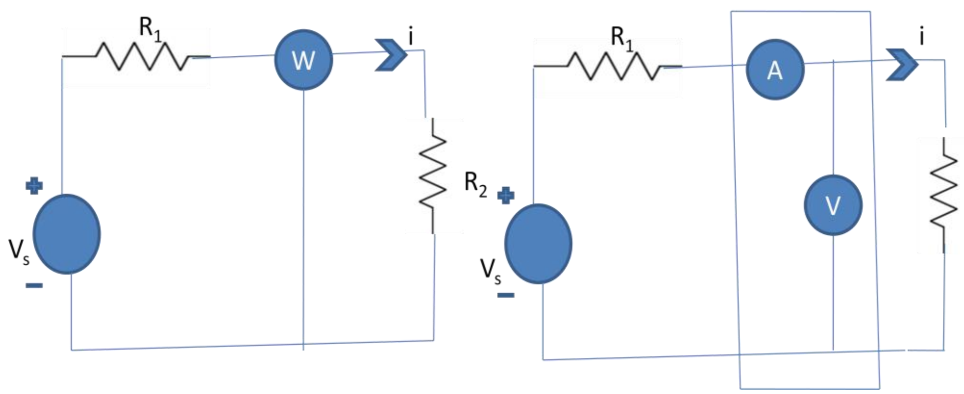
It measures voltage across a circuit element. Since voltage is the difference in potential between two points in a circuit. The voltmeter needs to be connected across the element whose voltage is to be measured. An ideal voltmeter has infinite internal resistance.

Symbolized as: 

(d) Watt meter/Power-factor meter:

It is the combination of voltmeter and ammeter which measures the power of the circuit.

Symbolized as: 



RESISTIVE NETWORK ANALYSIS:

This chapter illustrates the fundamental techniques for the analysis of resistive circuits. Let's first gain a brief idea about the different electric circuits and circuit parts given below:

1. **Circuit:** A circuit is a closed conducting path through which an electric current either flows or intended to flow. A circuit consists of active and passive elements in it.

2. **Linear Circuit**: A circuit whose parameters are constant with time and they don't change with voltage or current and they obeys Ohm's law.

3. **Non-linear Circuit**: It is that circuit whose parameters change with voltage or current.

4. **Bilateral Circuit:** A bilateral circuit is one whose properties are the same in either direction. Example, a transmission line & parameter such as resistor, inductor & capacitor.

5. **Unilateral Circuit:** It is a circuit whose properties change with the direction of its operation. Example such as diode, triode , transistor & SCR.

6. **Active Network:** is a network which contains one or more than one source of emf along with passive element.

7. **Passive Network:** is one which contains no source of emf.

8. **Node**: is a point in a circuit where two or more circuit elements are connected together.

9. **Branch**: is the part of a network which lies between two nodes.

10. **Loop:** It is a close path in a circuit in which no element or node is encountered more than once.

11. **Mesh**: It is a loop that contains no other loop within it.

It should be noted that, unless stated otherwise, an electric network would be assumed passive. There are two general approaches to network analysis:

i. **Direct Method**

ii. **Network Reduction Method**

Direct Method: Here the network is left in its original form while determining its voltages and currents.

e.g.: Loop Analysis, Nodal Analysis, Superposition Theorem, Compensation Theorem, and Reciprocity Theorem.

Network Reduction Method: Here, the original network is converted into a much simpler equivalent circuit for rapid and easy calculation.

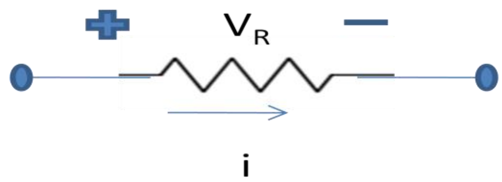
e.g.: Delta/Star and Star/delta Conversions, Thevenin’s Theorem and Norton’s Theorem.

MESH CURRENT ANALYSIS:

- Here mesh current is the independent variable.

- Kirchhoff's voltage law (KVL) is applied here around the mesh provides the desired system of equations.

- In this method, current flowing through a resistor in a specific direction defines the polarity of the voltage across the resistor and sum of the voltage around a closed circuit must be equal to 0.



- No of equation obtained by this method is equal to the no. of meshes in the circuit.

- There should be a specific direction of current in all the meshes i.e., either in clockwise or anticlockwise direction.

- No. of mesh equation, m= b-(j-1), where b= no. of branches

  j= no. of junctions

Consider a given network as shown in Fig.20.

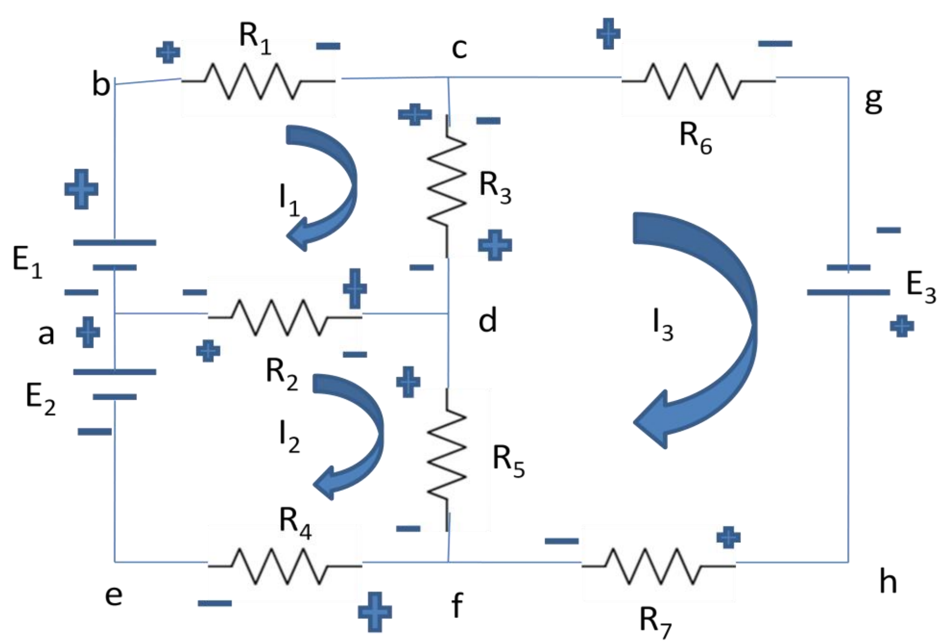


Fig.20 Mesh analysis of a circuit

The above network consists of resistances and independent voltage sources and has three meshes. Suppose the three meshes currents are designated as I_1, I_2 and I_3 and all are assumed to flow in a same direction i.e., clockwise direction.

No. of nodes: 8 (a, b, c, d, e, f, g, h)

No. of junction points: 3 (c, d, f)

No. of branches: 5 (cbad, cd, deaf, df, cghf)

So, no. of mesh equations: b-(j-1)= 5-(3-1)= 3

Applying KVL in mesh-1,

$$E_1 - I_1R_1 - R_3 (I_1 - I_3) - R_2 (I_1 - I_2) = 0$$

$$\text{Or, } (R_1+R_2+R_3) I_1 - R_2I_2 - I_3R_3 = E_1 \qquad \text{(i)}$$

Applying KVL in mesh-2,

$$E_2 - R_2 (I_2 - I_1) - R_5 (I_2 - I_3) - I_2R_4 = 0$$

$$\text{Or, } -R_2I_1 + (R_2+R_4+R_5) I_2 - R_5I_3 = E_2 \qquad \text{(ii)}$$

Applying KVL in mesh-3,

$$-R_6I_3 + E_3 - R_7I_3 - R_5 (I_3 - I_2) - R3 (I_3 - I_1) = 0$$

$$\text{Or,} -R_3I_1 - R_5I_2 + (R_3+R_5+R_6+R_7) I_3 = E_3 \qquad \text{(iii)}$$

It should be noted that signs of different terms in the above three equations have been so changed as to make the items containing self resistances positive. The matrix equivalent of these three equations is,

$$\begin{bmatrix} +(R1+R2+R3) & -R2 & -R3 \\ -R2 & +(R2+R4+R5) & -R5 \\ -R3 & -R5 & +(R3+R5+R6+R7) \end{bmatrix} \begin{bmatrix} I1 \\ I2 \\ I3 \end{bmatrix} = \begin{bmatrix} E1 \\ E2 \\ E3 \end{bmatrix}$$

It can be seen that, the first positive term in the first row is $(R_1+R_2+R_3)$ which represents the self resistance of mesh 1 which is equal to the sum of all resistance in mesh 1. Similarly, the second term in first row represents the mutual or common resistance between mesh 1 and mesh 2 i.e., the negative sum of the resistances common to mesh 1 and mesh 2. Similarly, the third term in this row represents the mutual resistance of mesh1 and mesh 3.

E_1 represents the algebraic sum of the voltages of all the voltage sources acting around mesh1. Similar is the case with E_2 and E_3. But the main thing is the sign of e.m.fs. While going along the current, if we pass from negative to the positive terminal of a battery, then its e.m.f is taken positive. If it is other way around, then battery emf is taken negative.

So, the general form of above matrix is as follows,

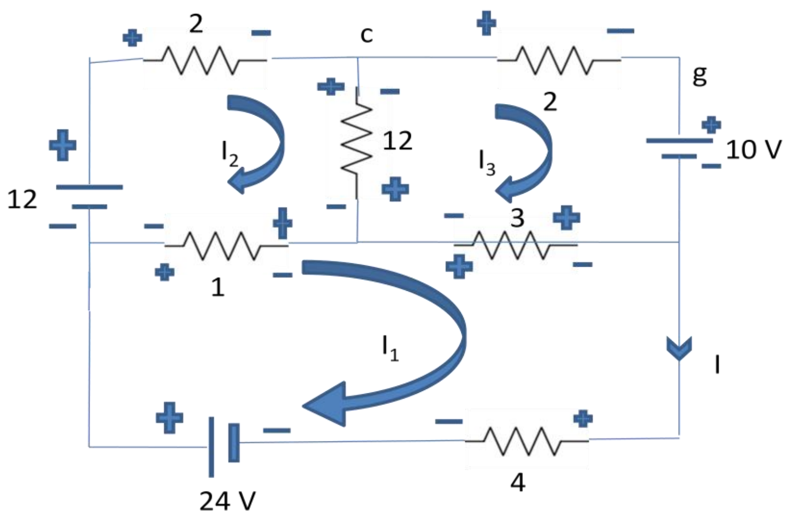
$$\begin{bmatrix} R11 & R12 & R13 \\ R21 & R22 & R23 \\ R31 & R32 & R33 \end{bmatrix} \begin{bmatrix} I1 \\ I2 \\ I3 \end{bmatrix} = \begin{bmatrix} E1 \\ E2 \\ E3 \end{bmatrix}$$

This above matrix is called "Mesh Resistance Matrix". If we choose each mesh current to flow in the clockwise direction. Then

- All the self resistance will always be positive.
- All mutual or common resistance will always negative.
- If two currents through a common resistance flow in the same direction, then the mutual resistance is taken as positive, opposite direction taken as negative.
- Arbitrary direction of current may be assumed for any current in the circuit; if the resulting numerical answer for that current is come out negative, then the chosen reference direction is opposite to the direction of actual current flow.

Example No.12

Determine the current in 4 Ω resistance in the circuit given below.



Solution: For loop 1,

$-1 (I_1- I_2) – 3 (I_1- I_3) – 4I_1 +24=0$

Or, by simplifying it will be $8I_1 –I_2 – 3I_3 = 24$ (i)

For loop 2,

$12- 2I_2 – 12(I_2- I_3) – 1 (I_2-I_1) =0$

Or, $I_1- 15I_2+ 12I_3= -12$ (ii)

For loop 3,

$-12 (I_3- I_2) -2 I_3-10-3(I_3- I_1) =0$

Or, $3I_1+12I_2-17I_3= 10$ (iii)

Solution by Determinant method:

The matrix form of above three equations is,

$$\begin{bmatrix} 8 & -1 & -3 \\ 1 & -15 & 12 \\ 3 & 12 & -17 \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \\ I_3 \end{bmatrix} = \begin{bmatrix} 24 \\ -12 \\ 10 \end{bmatrix}$$

I_1 is the current flowing through 4 Ω resistance. So,

$I_1 = \frac{\Delta 1}{\Delta}$

$$\Delta = \begin{bmatrix} 8 & -1 & -3 \\ 1 & -15 & 12 \\ 3 & 12 & -17 \end{bmatrix} = 664$$

$$\Delta_1 = \begin{bmatrix} 24 & -1 & -3 \\ -12 & -15 & 12 \\ 10 & 12 & -17 \end{bmatrix} = 2730$$

So, $I_1 = \frac{2730}{664} = 4.1$ A

Solution by Mesh Resistance Matrix:

R_{11}= 3+1+4=8 Ω; R_{22}= 2+12+1= 15 Ω; R_{33}= 2+3+12= 17 Ω

R_{12}= R_{21}= -1 Ω; R_{23}= R_{32}= -12 Ω; R_{13}= R_{31}= -3 Ω

E_1= 24 V; E_2= 12 V; E_3= -10 V

By writing in matrix form, we can get

$$\begin{bmatrix} 8 & -1 & -3 \\ -1 & 15 & -12 \\ -3 & -12 & 17 \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \\ I_3 \end{bmatrix} = \begin{bmatrix} 24 \\ 12 \\ -10 \end{bmatrix}$$

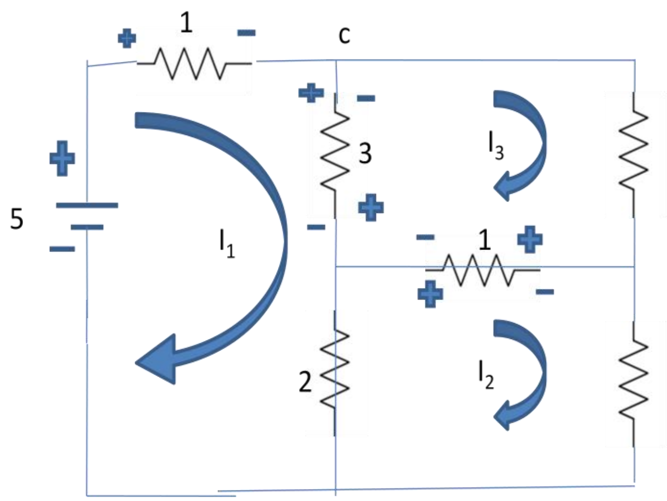
Δ= 664

$$\Delta_1 = \begin{bmatrix} 24 & -1 & -3 \\ 12 & 15 & -12 \\ -10 & -12 & 17 \end{bmatrix} = 2730$$

Current through 4 Ω resistance= $\frac{\Delta 1}{\Delta} = \frac{2730}{664} = 4.1A$

Example No.13

Find out the current I_3



Solution: In mesh-1, mesh equation is $6I_1$-$2I_2$-$3I_3$=5

In mesh-2, mesh equation is -$2I_1$+$7I_2$-I_3=0 &

In mesh-3, mesh equation is -$3I_1$-I_2 +$6I_3$=0

The mesh resistance matrix is,

$$\begin{bmatrix} 6 & -2 & -3 \\ -2 & 7 & -1 \\ -3 & -1 & 6 \end{bmatrix}\begin{bmatrix} I_1 \\ I_2 \\ I_3 \end{bmatrix} = \begin{bmatrix} 5 \\ 0 \\ 0 \end{bmatrix}$$

$$\Delta = \begin{bmatrix} 6 & -2 & -3 \\ -2 & 7 & -1 \\ -3 & -1 & 6 \end{bmatrix} = 147$$

$$\Delta_3 = \begin{bmatrix} 6 & -2 & 5 \\ -2 & 7 & 0 \\ -3 & -1 & 0 \end{bmatrix} = 115$$

$$I_3 = \frac{\Delta 3}{\Delta} = \frac{115}{147} = 0.782 \text{ A}$$

NODE VOLTAGE METHODE:

- The node voltage method is based on determining the voltage at each node as an independent variable. Here node voltage is the independent variable.
- One of the nodes is selected as a ‘reference node or datum node or zero potential node’. This node usually has most elements tied to it. All other nodes are referred or referenced to this node.

- Once each node voltage is determined, Ohm's law may be applied between any two adjacent nodes to find out current flowing in each branch.
- Each branch current is expressed in terms of one or more node voltages.
- KCL is applied in this node voltage method.
- Here a circuit of n nodes led to writing n-linear equations. But one of the nodes is to be referred as reference node and it is usually assumed to be 0. So, we can write (n-1) independent linear equation in the (n-1) independent variables.

Let's understand this with Fig.21 as shown below.

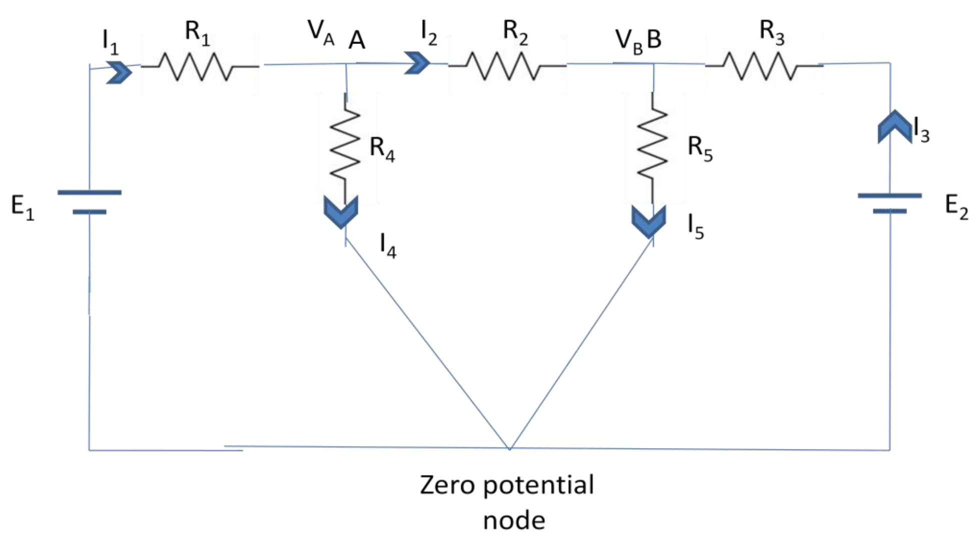


Fig.21 Node voltage Analysis

Applying KCL at node A, $I_1 = I_4 + I_2$

Now, $I_1 = \frac{E_1 - V_A}{R_1}$, $I_4 = \frac{V_A}{R_4}$, $I_2 = \frac{V_A - V_B}{R_2}$

So, substituting all values of I_1, I_2 and I_4,

$$\frac{E_1 - V_A}{R_1} = \frac{V_A}{R_4} + \frac{V_A - V_B}{R_2}$$

Simplifying the above, we got $V_A(\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_4}) - \frac{V_B}{R_2} - \frac{E_1}{R_1} = 0$

Applying KCL at node B, $I_5 = I_2 + I_3$

$$\text{Or, } \frac{V_B}{R_5} = \frac{V_A - V_B}{R_2} - \frac{E_2 - V_B}{R_3}$$

$$\text{Or, } V_B \left(\frac{1}{R_2} + \frac{1}{R_3} + \frac{1}{R_5} \right) - \frac{V_A}{R_2} - \frac{E_2}{R_3} = 0$$

From above, we can see that,

- The product of node potential V_A and $\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_4}$ i.e. the sum of the reciprocals of the branch resistance to this node.
- Minus the ratio of adjacent potential V_B and the interconnecting resistance R_2
- Minus ratio of adjacent battery voltage E_1 and interconnecting resistance R_1.
- All the above set to zero.

Second case, if there is a battery in between two nodes as shown in Fig.22 below, then

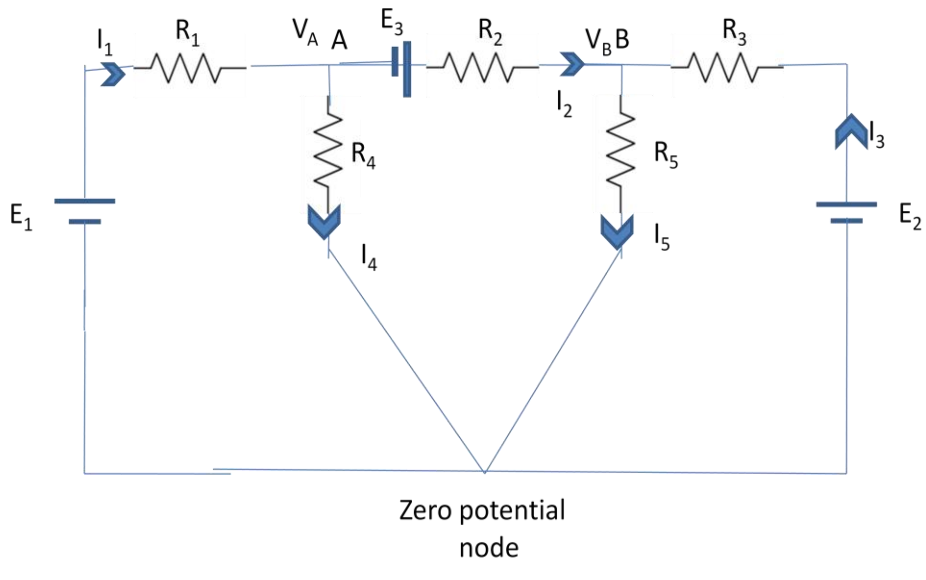


Fig.22

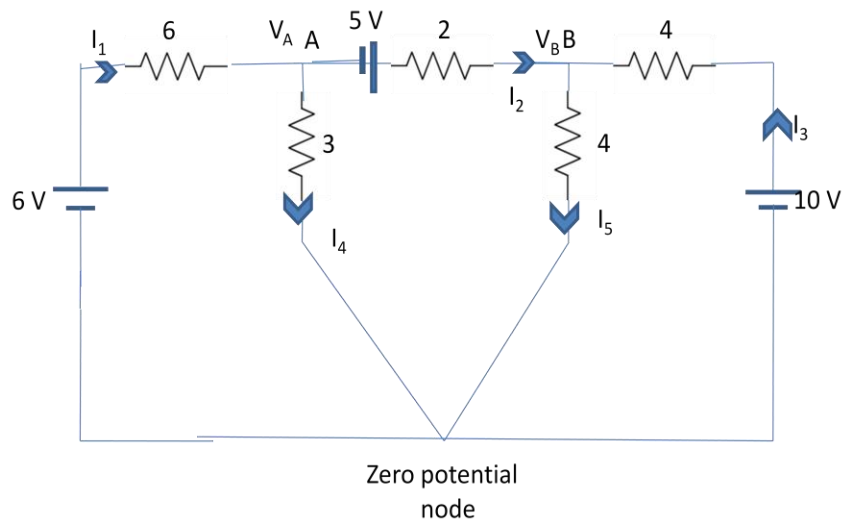
Here, a third battery E_3 is connected between node A and B. If we travel from node A to node B, we go from the negative potential of E_3 to its positive terminal. So, E_3 must be taken as positive. For node B, $I_5 = I_2 + I_3$

$I_2 = \frac{V_A + E_3 - V_B}{R_2}$, $I_3 = \frac{E_2 - V_B}{R_3}$, $I_5 = \frac{V_B}{R_5}$

$V_B\left(\frac{1}{R_2} + \frac{1}{R_3} + \frac{1}{R_5}\right) - \frac{E_2}{R_3} - \frac{V_A}{R_2} - \frac{E_3}{R_2} = 0$

Example No.14

Find the branch currents in the circuit by using nodal analysis.



Solution: The equation at node A can be written as,

$V_A\left(\frac{1}{6} + \frac{1}{2} + \frac{1}{3}\right) - \frac{6}{6} - \frac{V_B}{2} + \frac{5}{2} = 0$

Or, $2\,V_A - V_B = -3$ ------- (i)

At node B, $V_B\left(\frac{1}{2} + \frac{1}{4} + \frac{1}{4}\right) - \frac{10}{4} - \frac{V_A}{2} - \frac{5}{2} = 0$

Or, $V_B - \frac{V_A}{2} = 5$ --------- (ii)

From eq(i) and eq (ii), $V_A = \frac{4}{3}\ V$ and $V_B = \frac{17}{3}\ V$

$I_1 = \frac{E_1 - V_A}{R_1} = \frac{6-\frac{4}{3}}{6} = 7/9$ A

$I_2 = \frac{V_A + E_3 - V_B}{R_2} = \frac{\left(\frac{4}{3}\right)+5-\left(\frac{17}{3}\right)}{2} = \frac{1}{3}A$

$I_3 = \frac{E_2 - V_B}{R_3} = \frac{10-17/3}{4} = \frac{13}{12}A$

$I_4 = \frac{V_A}{R_4} = \frac{4/3}{3} = \frac{4}{9}\ A$

$I_5 = \frac{V_B}{R_5} = \frac{17/3}{4} = \frac{17}{12}\ A$

NODAL ANALYSIS WITH CURRENT SOURCES:

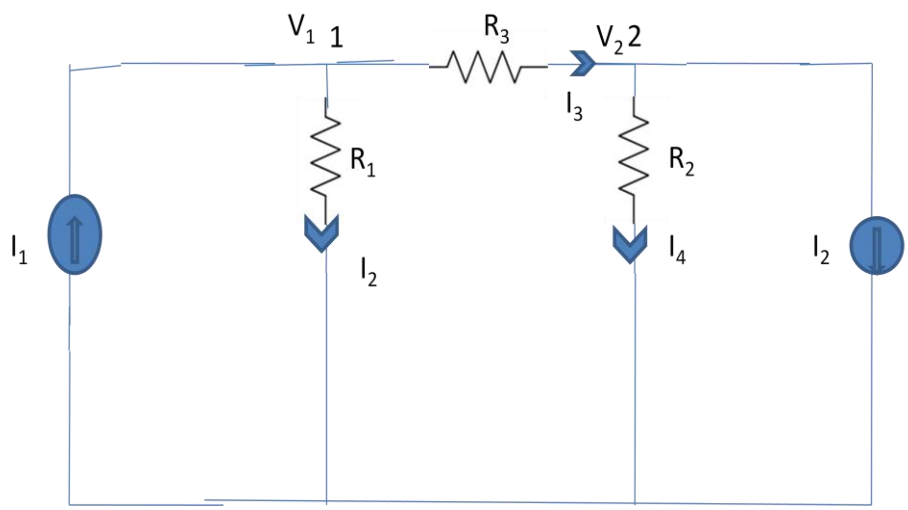


Fig.23 Node Voltage Analysis with Current Sources

Here, there are two current sources and two nodes. Both V_1 and V_2 are positive with respect to the reference node. That is why their respective currents flow from nodes 1 and 2 to the reference node. Current is flowing from node 1 to node 2, so V_1 is positive with respect to V_2 as shown in Fig.23.

Applying KCL at node 1,

$I_1 = I_2 + I_3$

Or, $I_1 = \frac{V_1}{R1} + \frac{V_1 - V_2}{R_3}$

Or, $V_1\left(\frac{1}{R_1} + \frac{1}{R_3}\right) - \frac{V_2}{R_3} = I_1$

Applying KCL at node 2: $I_3 = I_2 + I_4$

Or, $\frac{V_1 - V_2}{R_3} = I_2 + \frac{V_2}{R_2}$

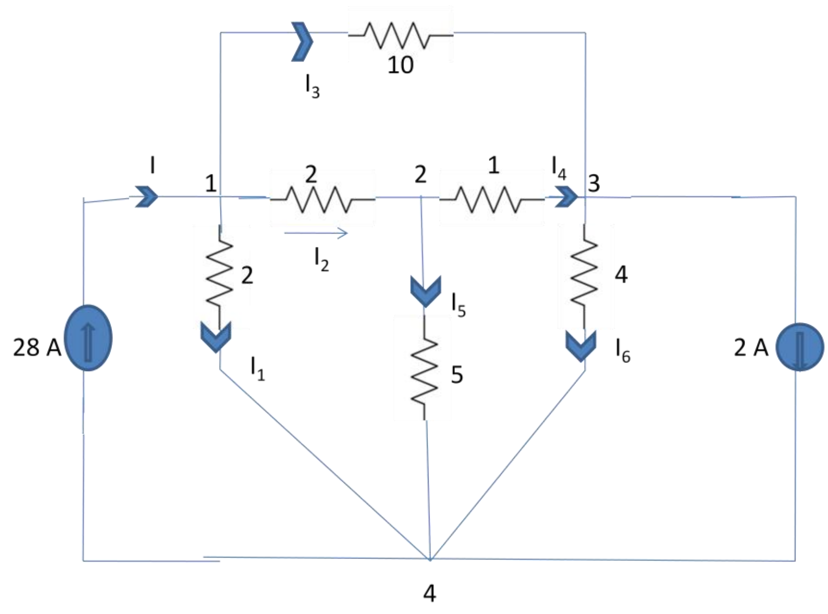
$$\text{Or,} \quad V_2\left(\frac{1}{R_2}+\frac{1}{R_3}\right)-\frac{V_1}{R_3}=-I_1$$

From above, we observed that,

- Product of potential V_1 and $\frac{1}{R_1}+\frac{1}{R_3}$ i.e., sum of the reciprocals of the branch resistances connected to this node
- Minus the ratio of adjoining potential V_2 and interconnecting resistance R_3
- All the above equated to the current supplied by the current source connected to this node.
- This current is taken as positive if flowing into the node and negative if flowing out of it.

Example No.15

Find the current in the various resistors in the circuits.



Solution: Node 1: $V_1\left(\frac{1}{2}+\frac{1}{2}+\frac{1}{10}\right)-\frac{V_2}{2}-\frac{V_3}{10}=8$

Or, $11\,V_1-5\,V_2-V_3=280$ (i)

Node 2: $V_2\left(\frac{1}{2}+\frac{1}{5}+1\right)-\frac{V_1}{2}-\frac{V_3}{1}=0$

Or, $5V_1 - 17V_2 + 10V_3 = 0$ (ii)

Node 3: $V_3\left(\frac{1}{4} + 1 + \frac{1}{10}\right) - \frac{V_2}{1} - \frac{V_1}{10} = -2$

Or, $V1 + 10V_2 - 13.5V_3 = 20$ (iii)

The matrix form of above equation is,

$$\begin{bmatrix} 11 & -5 & -1 \\ 5 & -17 & 10 \\ 1 & 10 & -13.5 \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \\ V_3 \end{bmatrix} = \begin{bmatrix} 280 \\ 0 \\ 20 \end{bmatrix}$$

$$\Delta = \begin{bmatrix} 11 & -5 & -1 \\ 5 & -17 & 10 \\ 1 & 10 & -13.5 \end{bmatrix} = 970$$

$$\Delta 1 = \begin{bmatrix} 280 & -5 & -1 \\ 0 & -17 & 10 \\ 20 & 10 & -13.5 \end{bmatrix} = 34{,}920$$

$$\Delta 2 = \begin{bmatrix} 11 & 280 & -1 \\ 5 & 0 & 10 \\ 1 & 20 & -13.5 \end{bmatrix} = 19{,}400$$

$$\Delta 3 = \begin{bmatrix} 11 & -5 & 280 \\ 5 & -17 & 0 \\ 1 & 10 & 20 \end{bmatrix} = 15{,}520$$

$V_1 = \frac{\Delta 1}{\Delta} = \frac{34920}{970} = 36\,V$, $V_2 = \frac{\Delta 2}{\Delta} = \frac{19400}{970} = 20\,V$, $V_3 = \frac{\Delta 3}{\Delta} = \frac{15520}{970} = 16\,V$

$I_1 = \frac{V_1}{2} = \frac{36}{2} = 18\,A$

$I_2 = \frac{V_1 - V_2}{2} = \frac{36-20}{2} = 8\,A$

$I_3 = \frac{V_1 - V_3}{10} = \frac{36-16}{10} = 2\,A$

$I_4 = \frac{V_2 - V_3}{1} = \frac{20-16}{1} = 4\,A$

$I_5 = \frac{V_2}{5} = \frac{20}{5} = 4\,A$

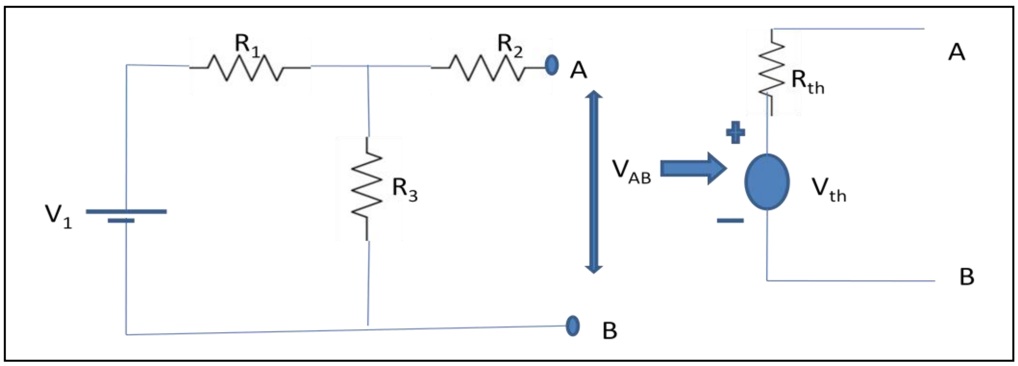
$I_6 = \frac{V_3}{4} = \frac{16}{4} = 4\,A$

THEVENIN'S THEOREM:

It provides a mathematical technique for replacing a given network as viewed from two output terminals, by a single voltage source with a series resistance.

- Any linear electrical network with voltage and current sources and only resistances can be replaced at terminals A-B by an equivalent voltage source V_{th} in series connection with an equivalent resistance R_{th}.
- This equivalent voltage V_{th} is the voltage obtained at terminals A-B of the network with terminals A-B open circuited.
- This equivalent resistance R_{th} is the resistance obtained at terminals A-B of the network with all its independent current sources open circuited and all its independent voltage sources short circuited.

This theorem also applies to frequency domain AC circuit consisting of reactive and resistive impedances.



Let's explain it briefly. Let's have a circuit as shown in Fig. 24 (a) below.

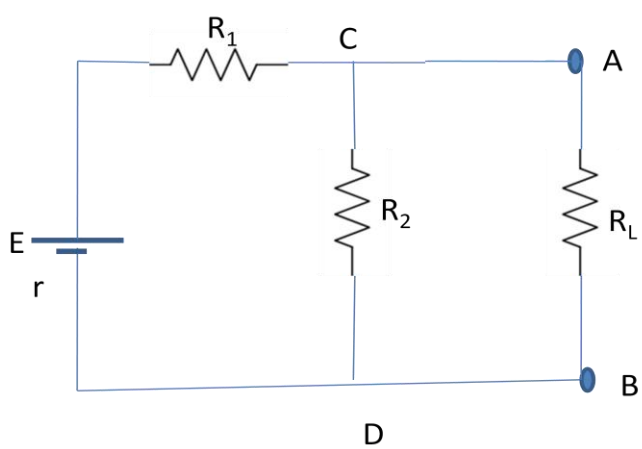


Fig. 24 (a)

Here we have to find out current through load resistances R_L. So, we will proceed as under,

1. First of all temporarily remove the resistance whose current is required. Here, R_L is the resistance whose current we have to find out. Redraw the circuit without load resistance as shown in Fig.24 (b).

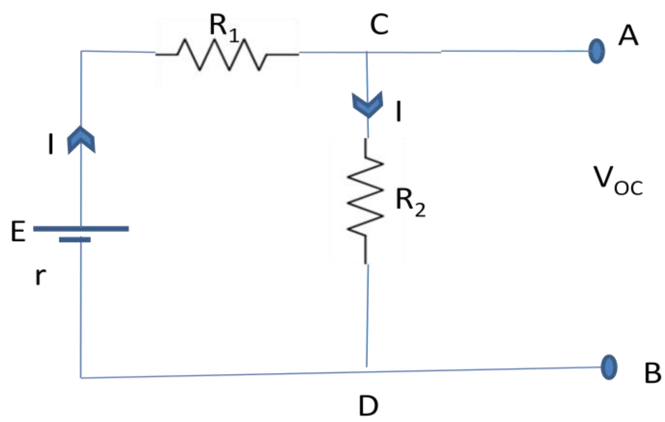


Fig. 24 (b)

2. Find the open-circuit voltage V_{OC} which appears across the two terminals from where resistance R_L has been removed. It is also called the Thevenin's Voltage, V_{th}.

V_{OC}= drop across R_2= IR_2, where I= circuit current when A and B are open.

$$I = \frac{E}{R_1 + R_2 + r}$$

$$V_{OC} = IR_2 = \frac{E\,R_2}{R_1 + R_2 + r} = V_{th}$$

3. Compute the resistance of the whole circuit as looked into from these two terminals after all voltage sources have been removed leaving behind their internal resistances (if any) and current sources have been replaced by open-circuit i.e. infinite resistance. It is also called Thevenin's resistance R_{th} as shown in FIg.24 (c).

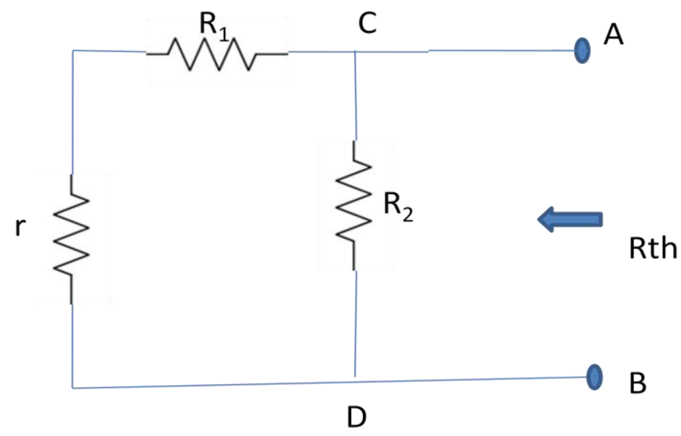


Fig. 24 (c)

R_{th}= $R_2 \parallel (R_1 + r) = \frac{R_2\,(R_1 + r)}{R_1+R_2+r}$

4. Replace the entire network by a single Thevenin source, whose voltage is V_{th} or V_{OC} and internal resistance R_{th}.

5. Connect R_L back to its terminals from where it was previously removed. Redraw the circuit again with V_{th} and R_{th} as shown in Fig.24 (d).

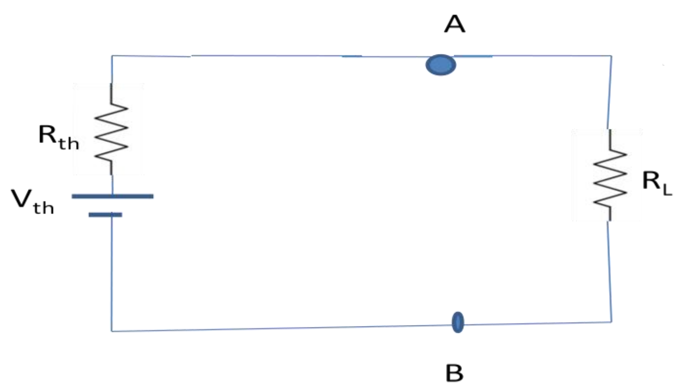
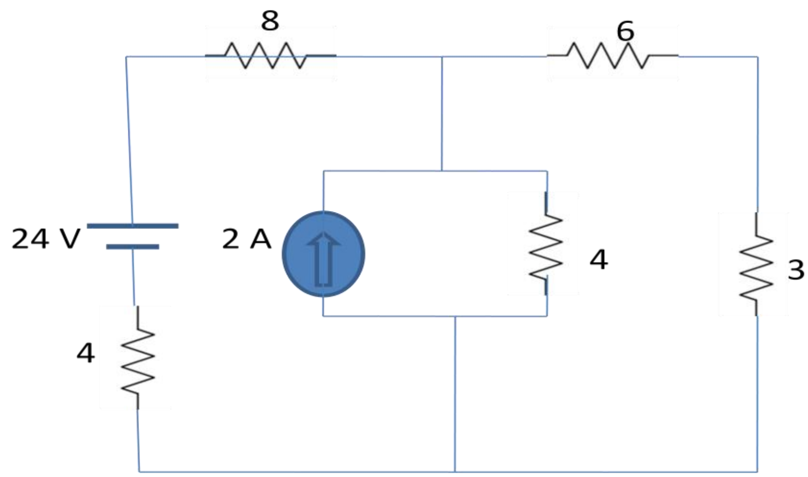


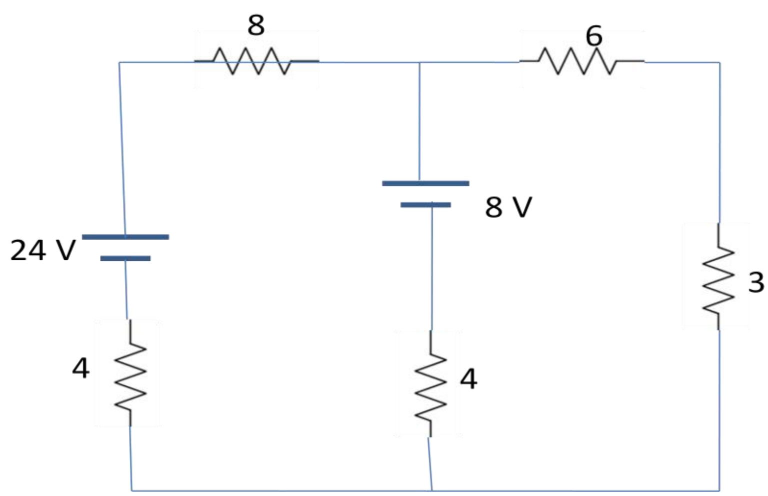
Fig. 24 (d)

Example No.16

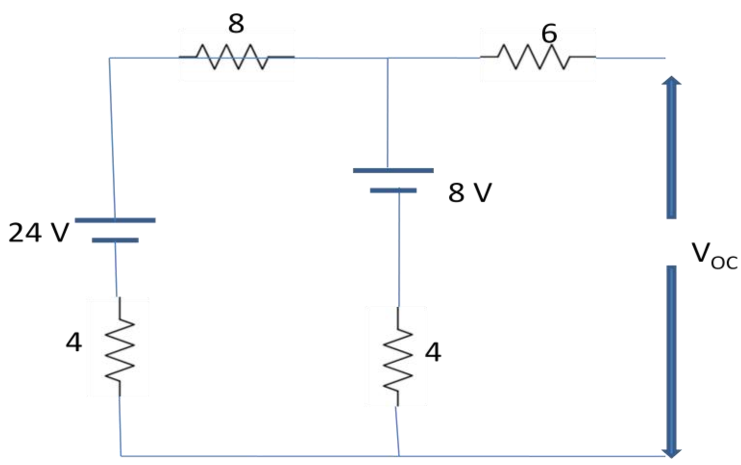
Calculate current flowing through the 3 Ω resistor.



Solution: First of all we have to convert 2 A current source and parallel resistance to a voltage source with series resistance by source conversion technique and redraw the circuit.



We have to find out current flowing through 3 Ω resistor. According to Thevenin's theorem, we have to remove that resistor whose current is required. So, 3 Ω resistor has to be removed from the circuit. After removing, the new circuit is as follows.



For finding V_{OC}, net voltage= 24- 8= 16 V

Total resistance= 8+4+4= 16 Ω

Current= 16/16= 1 A

So,Voltag drop across 4 Ω resistor= 4x1= 4 V in series with 8 V battery.

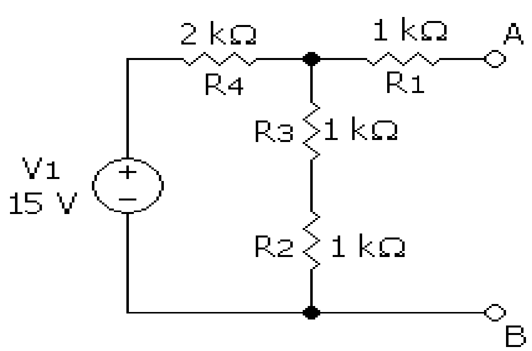
$V_{OC} = V_{th} = 8+4= 12$ V

For R_{th}, 6+4 || (8+4) = 9 Ω

I= $\frac{V_{th}}{R_{th}+R_L} = \frac{12}{9+3} = 1\ A$

Example No.17

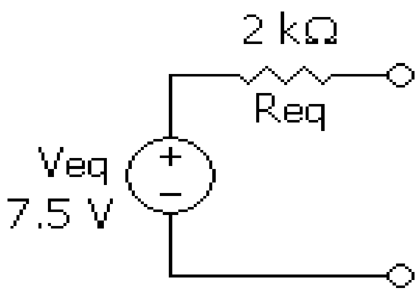
Find out the equivalent Thevenin's voltage and Thevenin's resistance of the circuit.



Solution: $V_{\text{Th}} = \frac{R_2 + R_3}{(R_2 + R_3) + R_4} \cdot V_1 = \frac{1+1}{1+1+2}.15 = 7.5\,V$

$R_{th} = R_1 + [(R_2 + R_3) \parallel R_4] = 2\Omega$

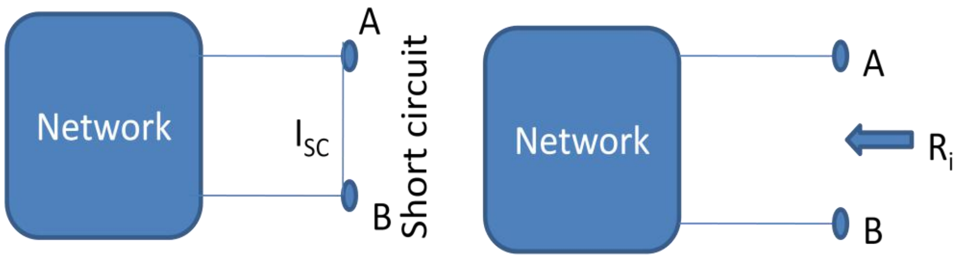
So, the obtained circuit is,



NORTON'S THEOREM:

This theorem is an alternative to the Thevenin's theorem. It is the dual of Thevenin's theorem. Here the two terminal network can be replaced by an equivalent constant current source and parallel resistor as shown in Fig.25. This theorem can be stated as follows,

"Any two-terminal active network containing voltage source and resistance when viewed from its output terminals is equivalent to a constant – current source and parallel resistance. The constant current is equal to the current which would flow in a short-circuit placed across the terminals and parallel resistance is the resistance of the network when viewed from these open circuited terminals after all voltage and current sources have been removed and replaced by their internal resistances."



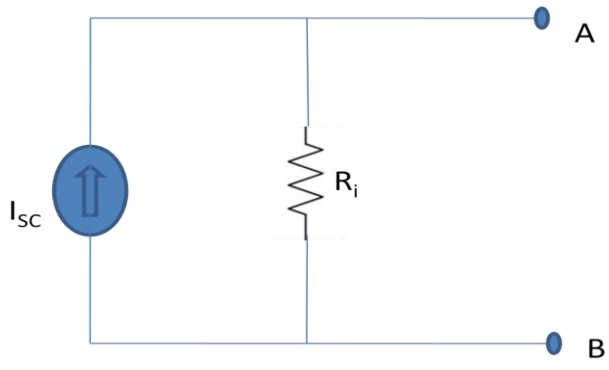
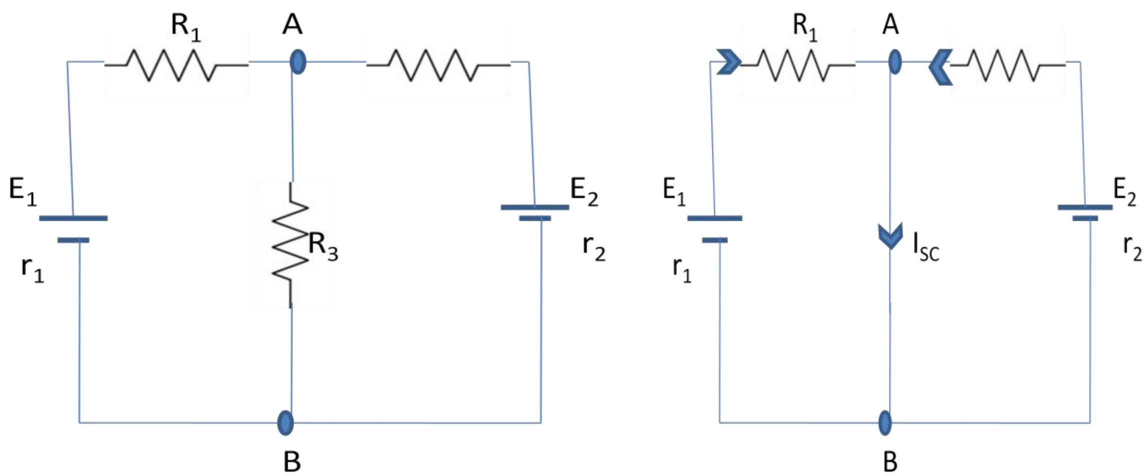


Fig.25 Norton equivalent circuit

The procedure to Nortonize a circuit is as follows,

1. Remove the resistance across the two given terminals and put a short- circuit across them.

It is required to find out voltage across R_3 and hence current through it. If short – circuit is placed between A and B, then current in it due to battery of emf E_1 is E_1/ R_1 and due to other battery is E_2/ R_2.



$$I_{SC} = \frac{E_1}{R_1} + \frac{E_2}{R_2} = E_1 G_1 + E_2 G_2$$

2. Internal resistance of the network as viewed from A and B by considering three resistances R_1, R_2 and R_3 connected parallel between A and B. Remove all voltage sources but retain their internal resistance, if any. Similarly remove all current sources and replace them by open-circuit. Now find resistance R_i of the network as looked from the given terminal.

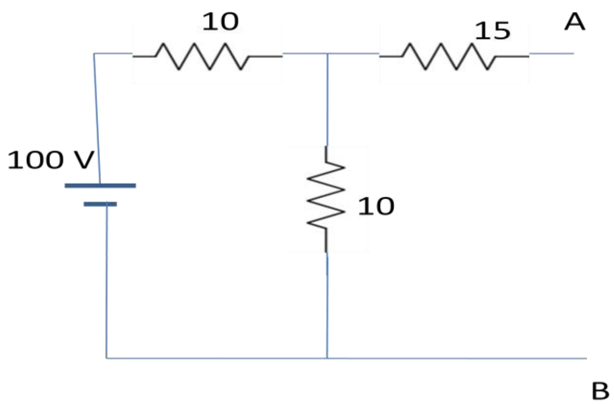
$$\frac{1}{R_i} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} = G_1 + G_2 + G_3$$

$R_i = \frac{1}{G_1+G_2+G_3}$

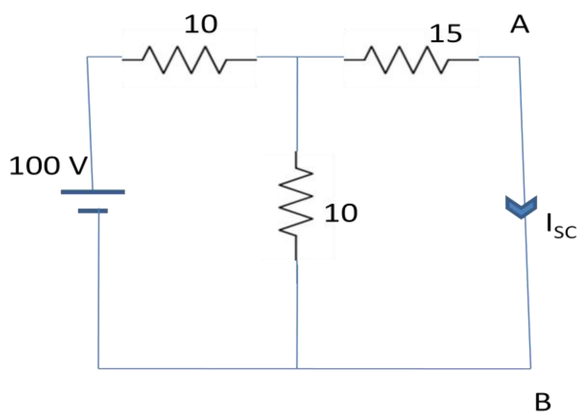
3. The current source I_{SC} joined in parallel across R_i between the two terminals gives Norton's equivalent circuit.

Example No.18

Convert the given circuit into Norton equivalent circuit.



Solution: First put a short circuit across AB terminals.



Total resistance= 10+ $\frac{15 \times 10}{15+10} = 16\ \Omega$

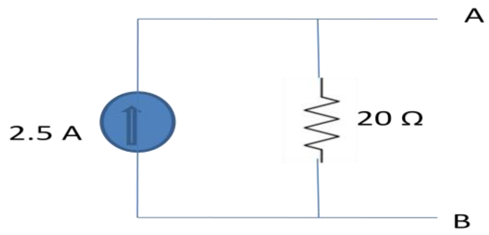
Battery current I= 100/16= 6.25 Ω

So, I_{SC}= 6.25× 10/25= 2.5 A

Since the battery has no internal resistance, the input resistance of the network when viewed from A and B consists of a 15 Ω resistance in series with parallel combination of 10Ω and 10Ω.

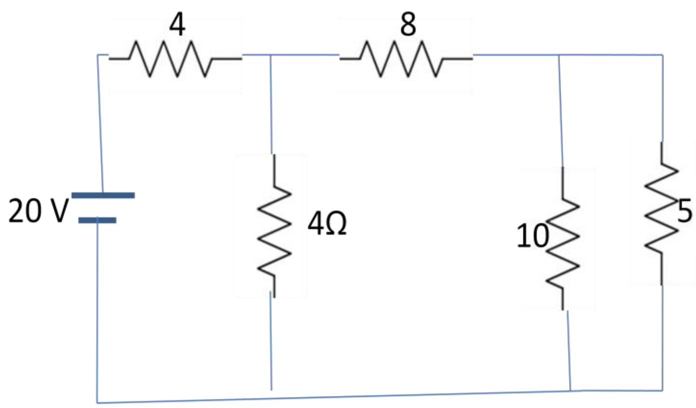
$R_i = 15 + (10/2) = 20\Omega$

The Nortonised circuit is,

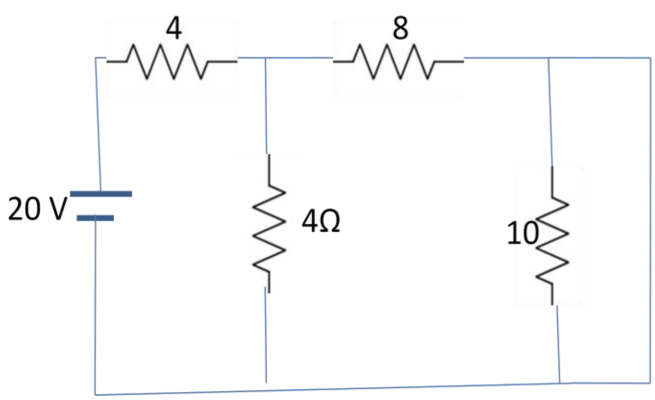


Example No.19

Find current through 5 Ω resistors.



Solution: Remove 5 Ω resistor and put short circuit across it.



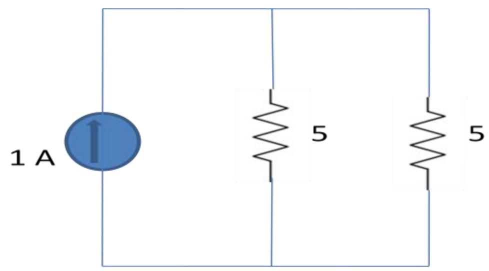
Here 10 Ω resistor also become short circuited.

Total resistance of the circuit= 4 +(4 ‖ 8)= 20/3 Ω

$I = \frac{20}{20/3} = 3$ A

So, $I_{SC} = 3 \times \frac{4}{4+8} = 1$ A

For finding resistance of the circuit R_i= 10 ‖ 10= 5Ω. The Norton equivalent circuit is



IDEAL CONSTANT VOLTAGE SOURCE:

It is that voltage source whose output voltage remains absolutely constant whatever the change in load current. Here voltage source must possess zero internal resistance. So that internal voltage drop in the source is zero. Output voltage provided by the source would remain constant irrespective of the amount of current drawn from it. The figure of ideal constant voltage source is shown in Fig.26.

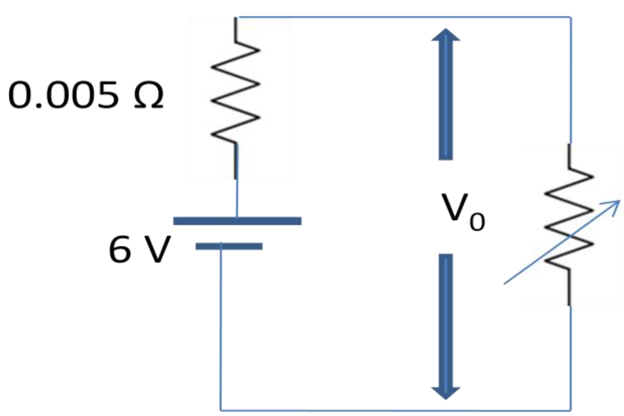


Fig.26 Ideal Constant Current Source

6 V battery with internal resistance 0.005 Ω.

- When no current i.e. at no-load; V_0= 6 V (same as supply)

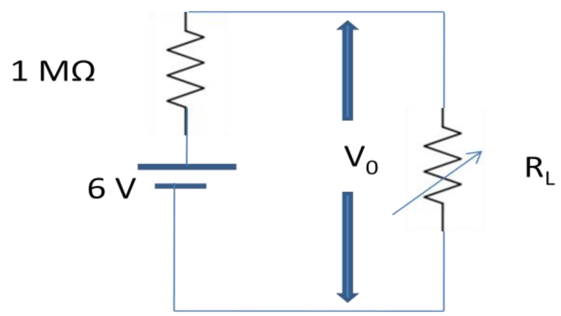
- If load current= 100 A; V_0= 100* 0.005= 0.5 V

  V_0= 6- 0.5= 5.5 V

So from above, output voltage of 5.5-6 V can be considered constant as compared to wide variation in load current from 0 A to 100 A.

IDEAL CONSTANT CURRENT SOURCE:

It is that that voltage source whose internal resistance is infinite. Here, source possesses very high resistance as compared to that of external load.



6 V battery has an internal resistance of 1 MΩ and load resistance varies between 20 KΩ to 200 KΩ. So, current supplied by the source varies from 6/1.02 = 5.9 μA to 6/1.2 = 5 μA. So, it is seen that even when load resistance increases 10 times, current decreases by 0.9 μA. Hence source can be considered constant current source.

SUPERPOSITION PRINCIPLE:

In a network of linear resistances containing more than one source of emfs, the current which flows at any points is the sum of all the current which would flow at that point if each emfwhere considered separately and all the other emfs are replaced by internal resistances for the timing.

Or, The superposition theorem states that in any network containing more than one source, the current in or the potential difference across any branch can be found out by considering each source separately and adding their effects; omitted sources of emf are replaced by resistances equal to their internal resistances. It can be explained with the help of Fig. 26 (a).

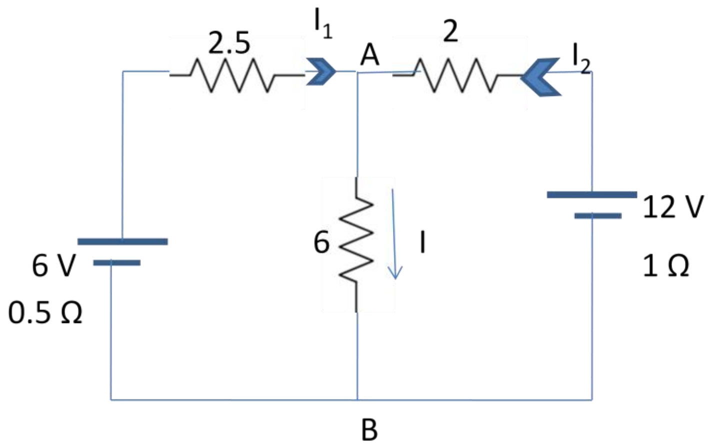
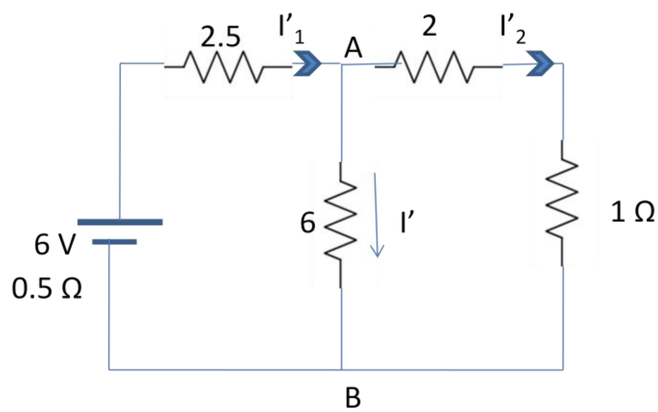


Fig.26 (a)

While considering 6 V only as shown in figure 26 (b):



Here only 6 V is considering by removing 12 V with its internal resistance of 1 Ω.

Equivalent resistance= 0.5+ (3|| 6) +2.5= 5 Ω

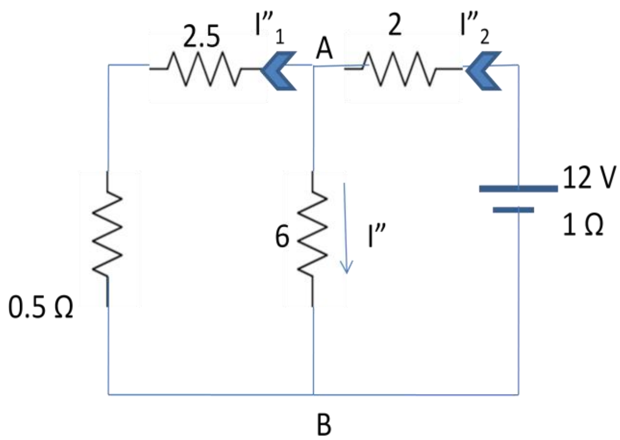
I'_1= 6/5= 1.2 A

This current divides at point A inversely in the ratio of the resistances of the two parallel paths.

$I' = 1.2 \times \frac{3}{9} = 0.4$ A

$I'_2 = 1.2\times \frac{6}{9} = 0.8$ A

While considering 12 V only as shown in Fig.24 (c):



Here only 12 V is considering by removing 6 V battery with its internal resistance of 0.5 Ω.

Equivalent resistance= $2+(6 \parallel 3) = 5\ \Omega$

$I''_2 = 12/5 = 2.4$ A

Applying current division rule, $I'' = 2.4 \times \frac{3}{9} = 0.8\ A$

$$I''_1 = 2.4 \times \frac{6}{9} = 1.6\ A$$

The actual current values can be obtained by superposition of these two sets of current values.

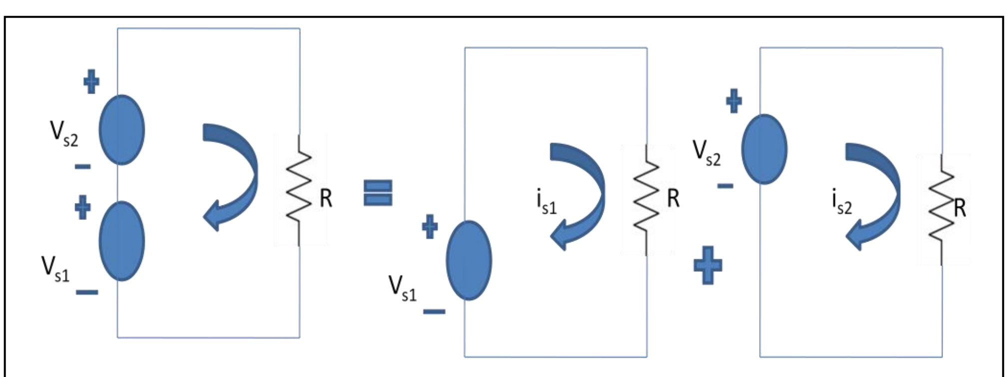
$I_1 = I'_1 - I''_1 = 1.2 - 1.6 = -0.4$ A (it is a charging current.)

$I_2 = I''_2 - I'_2 = 2.4 - 0.8 = 1.6$ A

$I = I' + I'' = 0.4 + 0.8 = 1.2$A

Note:

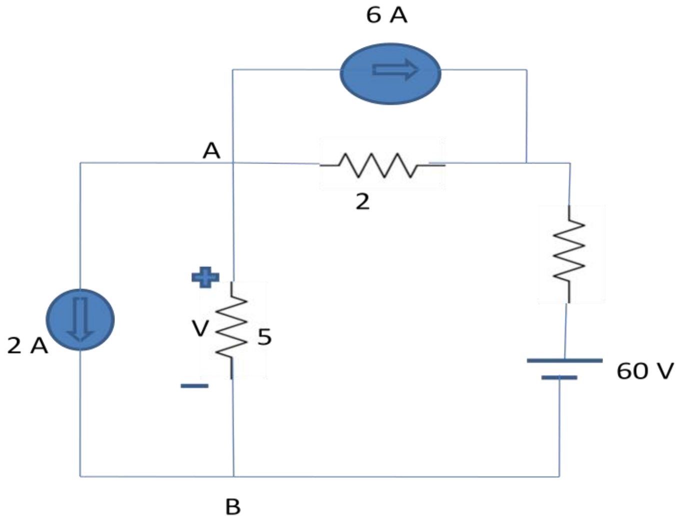
- In order to set a voltage source equal to zero, we replace it with a **short circuit**.
- In order to set a current source equal to zero, we replace it with **open circuit**.



According to superposition theorem, one source should be present at a time, deactivating remaining sources. Contributions due to individual sources are finally algebraically added to get the final answer.

Example No.20

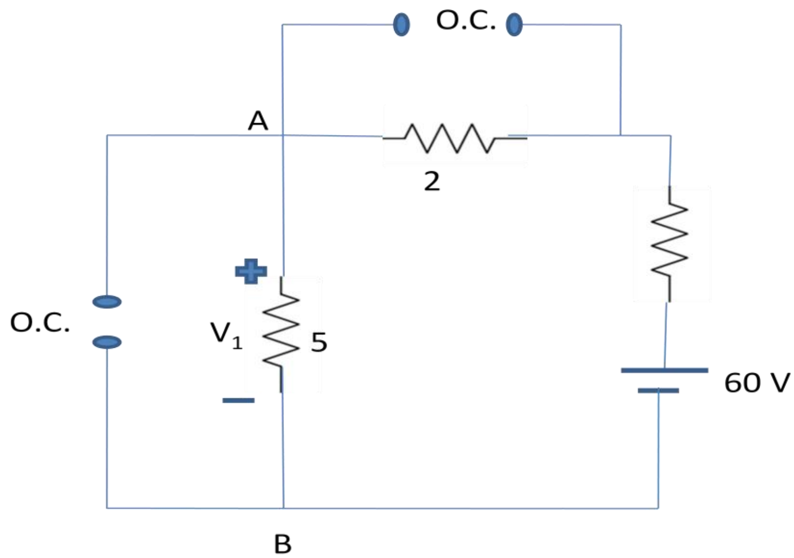
Applying superposition theorem, find the voltage drop across 5 Ω resistor.



Solution:

While taking 60 V only :

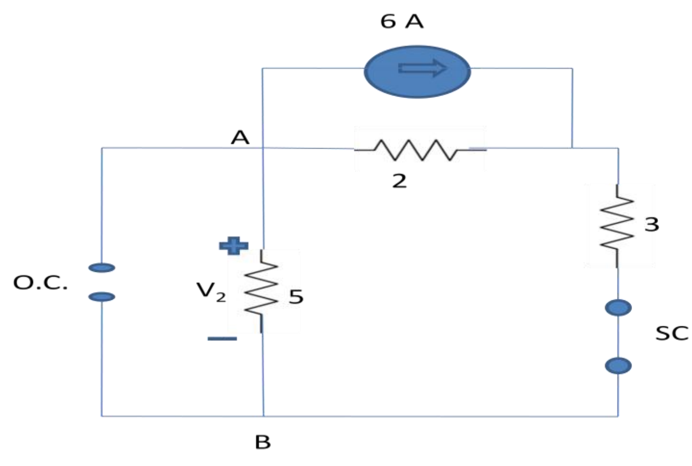
Here 60 V is first taking into account by leaving two current sources i.e., 2 A and 6 A with open circuited. So, the redrawn circuit is as follows,



$V_1 = 60\times\frac{5}{5+2+3} = 30\ V$; its value is taken positive, because current through the 5 Ω resistor from A to B, so making the upper end of resistor positive and the lower end negative.

While taking 6 A current source only :

Here 6 A current source is taking into account alone by leaving 60 V source as short circuit and 2 A current source as open circuit. The redrawn circuit is,



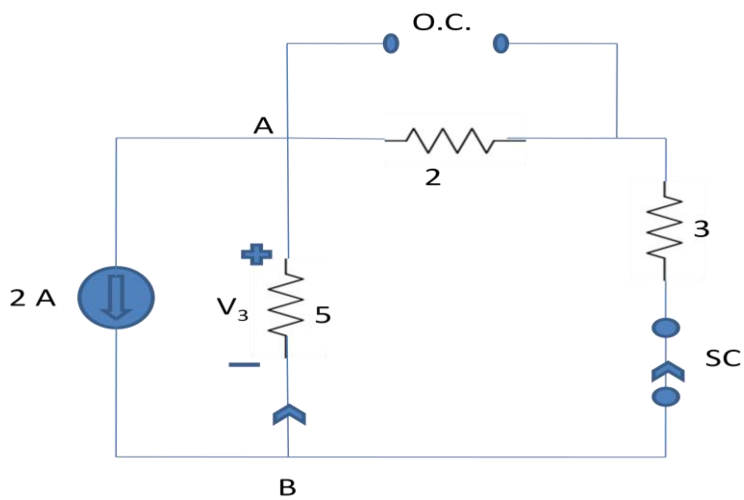
In the above figure, the 6 A current source has two parallel circuit across it; one having the resistor of 2Ω and other path having the series connection of resistances (3+5) = 8 Ω. Using the current divider rule, the current through 5 Ω resistor= $6\times\frac{2}{(2+3+5)} = 1.2$ A

V_2= 1.2* 5= 6 V, it would be taken negative because current is flowing from B to A i.e., point B is at higher potential as compared to point A.

V_2= - 6 V

While taking 2 A current source only :

Here only 2 A current source is taking into account while the other sources are dead.



Current 2 A divides equally at point B, because the two parallel paths have equal resistance of 5 Ω each.

V_3= 5* 1= 5 V. It would be taken negative because current flows from B to A.

Hence, V_3= - 5 V.

Using superposition principle, we got,

$$V= V_1+V_2+V_3= 30\text{-}6\text{-}5= 19\ V$$

MAXIMUM POWER TRANSFER THEOREM:

Although applicable to all electrical networks, this theorem is particularly useful for analyzing communication networks. The overall efficiency of a network is 50 percent. As applied to dc networks, this theorem may be stated as follows,

“A resistive load will abstract maximum power from a network when the load resistance is equal to the resistance of the network as viewed from the output terminal, with all energy sources removed leaving behind their internal resistances".

Fig.27 below shows that, a load resistance of R_L is connected across the terminals A and B of the network which consists of an emf of E and internal resistance R_g and a series resistance R.

Let, $R_i = R_g + R$ = internal resistance of whole network while viewing from AB terminals.

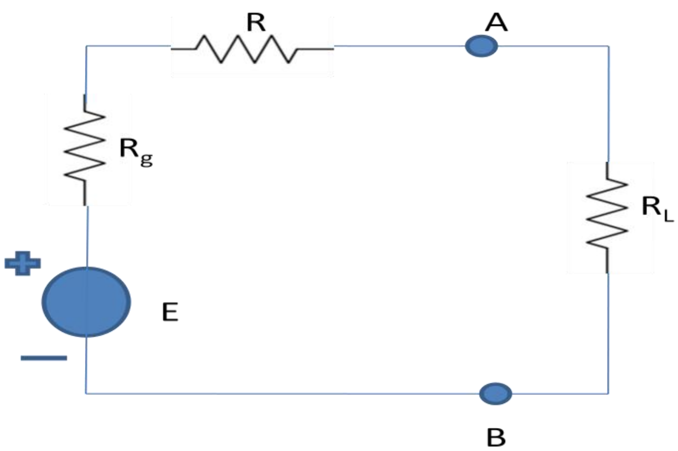


Fig. 27

According to this theorem, R_L will abstract maximum power from the network when $R_L = R_i$.

<u>Proof:</u> Circuit current $I = \frac{E}{R_L + R_i}$

Power consumed by load is $P_L = I^2 R_L = \frac{E^2 R_L}{(R_L + R_i)^2}$

For P_L to be maximum, $\frac{dP_L}{dR_L} = 0$

Differentiating we got, $2R_L = R_L + R_i$

Or, $R_L = R_i$

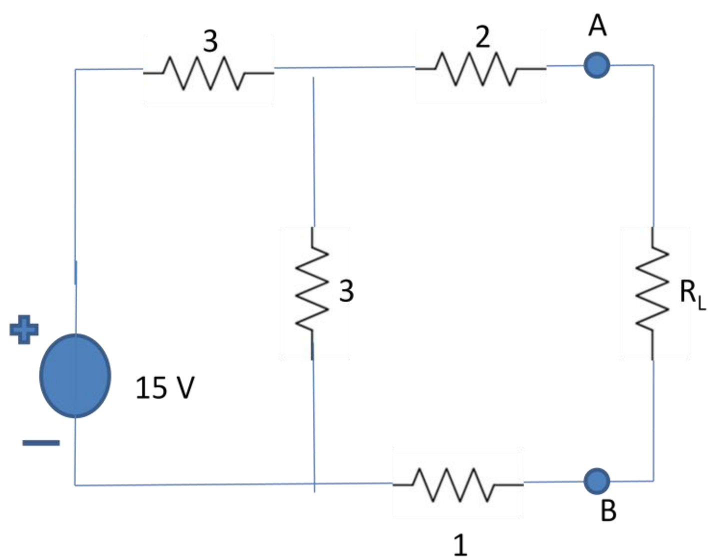
It should be noted that, under this condition the voltage across the load is hold the open circuit voltage at the terminal A and B.

Max. Power, $P_{LMax.} = \frac{E^2 R_L}{4\,R_L^2} = \frac{E^2}{4R_L} = \frac{E^2}{4R_i}$

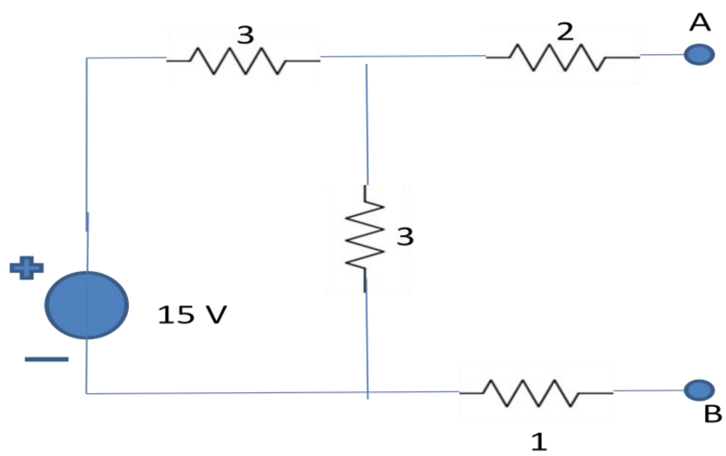
Note: An ac source of internal impedance (R_1+ jX_1) supplying power to the load impedance (R_L+jX_L), maximum power transfer can be taken place when, $| Z_L | = | Z_1 |$.

Example No.21

In the network shown below, find out the value of R_L such that maximum possible power will be transferred to R_L. Find also the value of the maximum power and the power supplied by source under these conditions.



Solution: First remove R_L to find the equivalent Thevenin's source of the circuit to the left of terminals A and B.



V_{th} is equal to the drop across 3 Ω resistor because no current will flow through 2 Ω and 1 Ω.

$V_{th} = \frac{15}{3+3} \times 3 = 7/5$ V

R_{th} is resistance found by replacing 15 V source with a short circuit.

$R_{th} = 2+(3\|\, 3)+1 = 4.5\ \Omega$

Maximum power transfer will take place when $R_L = R_{th} = 4.5\ \Omega$

Maximum power drawn by $R_L = \frac{V_{th}^2}{4} \times R_L = \frac{7.5^2}{4} \times 4.5 = 3.125\ W$

Power supplied by the source=2*3.125= 6.250 W

POWER TRANSFER EFFICIENCY:

If P_L is the power supplied to the load and P_T is the total power supplied by the voltage source, then power transfer efficiency given by

$$\eta = \frac{P_L}{P_T}$$

The voltage source E supplies power to both the load resistance R_L and to the internal resistance $R_i = (R_g+R)$

$$P_T = P_L + P_i \quad \text{or } E \times I = I^2 R_L + I^2 R_i$$

$$\eta = \frac{P_L}{P_T} = \frac{I^2 R_L}{I^2 R_L + I^2 R_i} = \frac{R_L}{R_L + R_i} = \frac{1}{1+(\frac{R_i}{R_L})}$$

The maximum value of η is unity when $R_L = \infty$ and has a value of 0.5 when $R_L = R_i$. It means under maximum power transfer conditions, the power transfer efficiency is only 50 %.

HOMEWORK PROBLEMS: